Naturalness and vacuum stability in type-II seesaw model

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Introduction

- Observed Higgs mass is ~125GeV
- Quantum correction for scalar field is given by

$$m_S^2(\Lambda,\mu) = \bar{m}_S^2(\Lambda) + \sum_{A=S,V,F} (-1)^{2J_A}(2J_A+1)\frac{g_A}{16\pi^2}\left[\Lambda^2 - \bar{m}_A^2(\Lambda)\ln\frac{\Lambda^2}{\mu^2}\right]$$

Λ: cutoff scale, μ: renormalization scale, $\bar{m}_A$: bare mass, $(\Lambda > \mu > m_A)$

J_A: spin, g_A: coupling betweein S and A (e.g. $g_F = y_F^2$ for fermion)

- There are two types of corrections
  - Quadratic divergence $(\propto \Lambda^2)$
  - Logarithmic divergence $(\propto \ln \Lambda^2/\mu^2)$

$\Lambda^2 \gg |m_S^2|$ seems to cause hierarchy problem.

fine-tuning between $\bar{m}_S^2$ and δm_S^2

Introduction

In fact, quadratic divergence is always subtracted.

['72 G.'t Hooft, M.J.G.Veltman], ['72 C.G.Bollini, J.J.Giambiagi] (Dimensional regularization)
['11 K.Fujikawa] (Subtraction scheme) ['12 H.Aoki, S.Iso] (Wilsonian renormalization)

⇨ We need consider only logarithmic divergence

Log. div. is understood by β-function of Higgs mass parameter.

RGE in the SM

$$\frac{dm_h^2}{d\ln\mu} = \frac{1}{16\pi^2} m_h^2 \left[12\lambda_H + 6y_t^4 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 \right]$$

$m_h^2(\mu) \sim m_h^2(\Lambda)$ Order of magnitude does not change

RGE in some extended SM

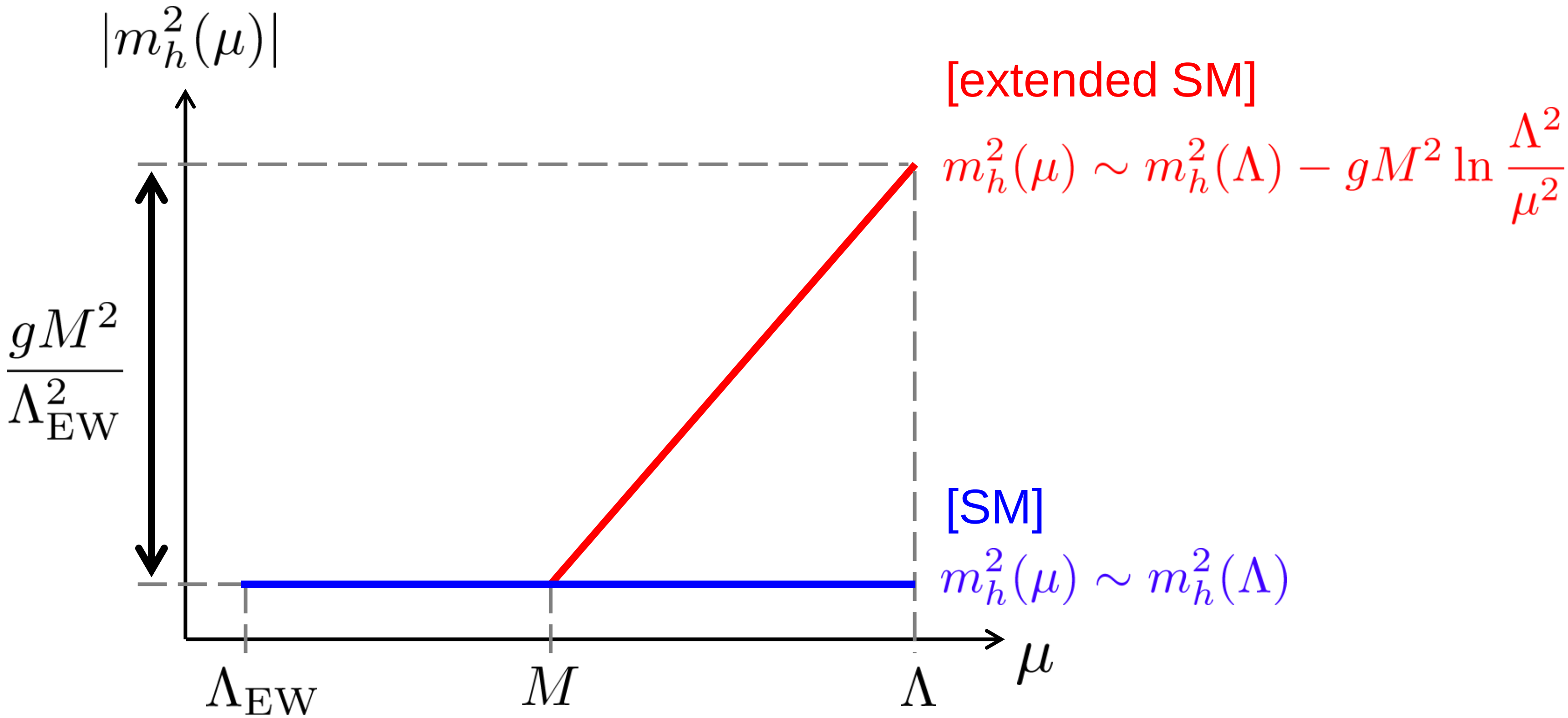
$$\frac{dm_h^2}{d\ln\mu} = \frac{1}{16\pi^2} m_h^2 \left[12\lambda_H + 6y_t^4 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 \right] + \frac{1}{16\pi^2} gM^2$$

$m_h^2(\mu) \sim m_h^2(\Lambda) - gM^2 \ln\frac{\Lambda^2}{\mu^2}$ Large contribution appears

$(\text{for } \mu > M)$

Introduction

Renormalization condition at the UV scale is sensitive to the heavy particle mass.



$$\text{Hierarchy problem arises only when } \frac{gM^2}{\Lambda_{\rm EW}^2} \gg 1.$$

Our study

The SM has to be extended because of

- active neutrino masses
- vacuum stability
- etc.

- We consider minimal type-II seesaw model
  - only triplet scalar field is added into the SM
- We investigate Vacuum stability and Perturbativity conditions
- In addition, to avoid the hierarchy problem, we also require Naturalness condition $(gM^2/\Lambda_{\rm EW}^2 \lesssim 1)$

Type-II seesaw model

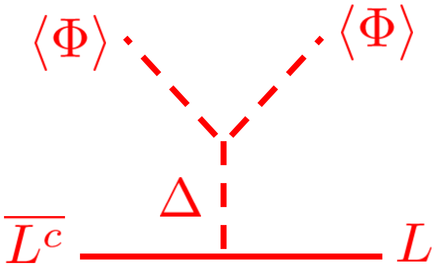
There are additional terms in Lagrangian:

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}}(Y_\Delta)_{ij} L_i^{\mathrm{T}} C i\sigma_2 \Delta L_j + \mathrm{H.c.}$$

$$V(\Phi,\Delta) = -m_\Phi^2 \Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right) + \lambda_4(\Phi^\dagger\Phi)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5\Phi^\dagger[\Delta^\dagger,\Delta]\Phi + \left(\frac{\lambda_6 M_\Delta}{\sqrt{2}}\Phi^{\mathrm{T}} i\sigma_2\Delta^\dagger\Phi + \mathrm{H.c.}\right)$$

['07 M.A.Schmidt] for this notation

where $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $\Delta = \frac{\sigma^i}{\sqrt{2}}\Delta_i = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$

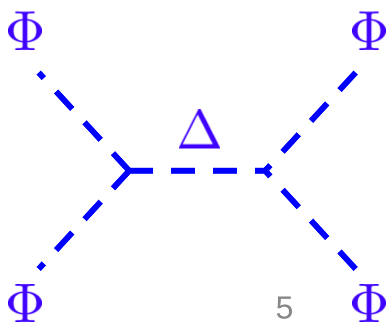


- Active neutrino masses

$$(M_\nu)_{ij} = v_\Delta (Y_\Delta)_{ij} \approx \frac{\lambda_6 M_\Delta v^2}{2M_\Delta^2 + v^2(\lambda_4 - \lambda_5)}(Y_\Delta)_{ij} \approx \frac{\lambda_6 v^2}{2M_\Delta}(Y_\Delta)_{ij}$$

- Low energy effective Higgs potential

$$V_{\mathrm{eff}} = -m_\Phi^2\Phi^\dagger\Phi + \frac{1}{2}\lambda_{\mathrm{SM}}(\Phi^\dagger\Phi)^2 \quad \text{with} \quad \lambda_{\mathrm{SM}} = \lambda - \lambda_6^2$$



Vacuum stability and perturbatibity

- Potential is bounded from below when ['11 A.Arhrib, et.al.]

$$\lambda \geq 0, \quad \lambda_1 \geq 0, \quad 2\lambda_1 + \lambda_2 \geq 0,$$

$$\lambda_4 + \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \quad \lambda_4 + \lambda_5 + \sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} \geq 0,$$

$$\lambda_4 - \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \quad \lambda_4 - \lambda_5 + \sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} \geq 0$$

In fact, corrections of these conditions have been recently pointed out in arXiv:1508.02323. [C.Bonilla, R.M.Fonseca, J.W.F.Valle]

The corrections does not significantly affect our results.

- We require all quartic couplings are less than 4π up to the Planck scale

Naturalness

Higgs mass corrections come from triplet scalar.

['07 A.Abada, C.Biggio, F.Bonnet, M.B.Gavela, T.Hambye]

$$\Rightarrow \quad \delta m_h^2 = -\frac{3}{16\pi^2}\left(\lambda_4 + \lambda_6^2\right) M_\Delta^2 \ln\frac{M_{\rm Pl}^2}{M_\Delta^2} \qquad (M_{\rm Pl} = 2.4\times 10^{18}\,{\rm GeV})$$

We require $\delta m_h^2 < M_h^2 = (125\,{\rm GeV})^2$

Other Naturalness condition

- Two-loop EW interaction: $M_\Delta \lesssim 200\,{\rm GeV}$

  ['13 M.Farina, D.Pappadopulo, A.Strumia]

- Veltman condition: $M_{H^\pm} < 288\,{\rm GeV},\ M_{H^{\pm\pm}} < 351\,{\rm GeV}$

  ['15 M.Chabab, M.C-Peyranere, L.Rahili]

Experimental bounds

- ρ-parameter ['12 PDG]

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1+\frac{2v_\Delta^2}{v^2}}{1+\frac{4v_\Delta^2}{v^2}} \implies v_\Delta \lesssim 5\,\mathrm{GeV}$$

- Neutrino Yukawa coupling

$$Y_\Delta = \frac{10^{-2}\,\mathrm{eV}}{v_\Delta} \times \mathcal{O}(1)_{3\times 3} < 1 \implies v_\Delta \gtrsim \mathcal{O}(10^{-2}\,\mathrm{eV})$$

- Same sign dilepton ['14 ATLAS]

$$H^{\pm\pm} \to \ell^\pm \ell^\pm \implies M_{H^{\pm\pm}} > 550\,\mathrm{GeV} \text{ for } v_\Delta < 10^{-4}\,\mathrm{GeV}$$

(large Yukawa region)

- Lepton flavor violating decay $(\mu \to e\gamma,\ \tau \to \bar{\ell}\ell\ell)$

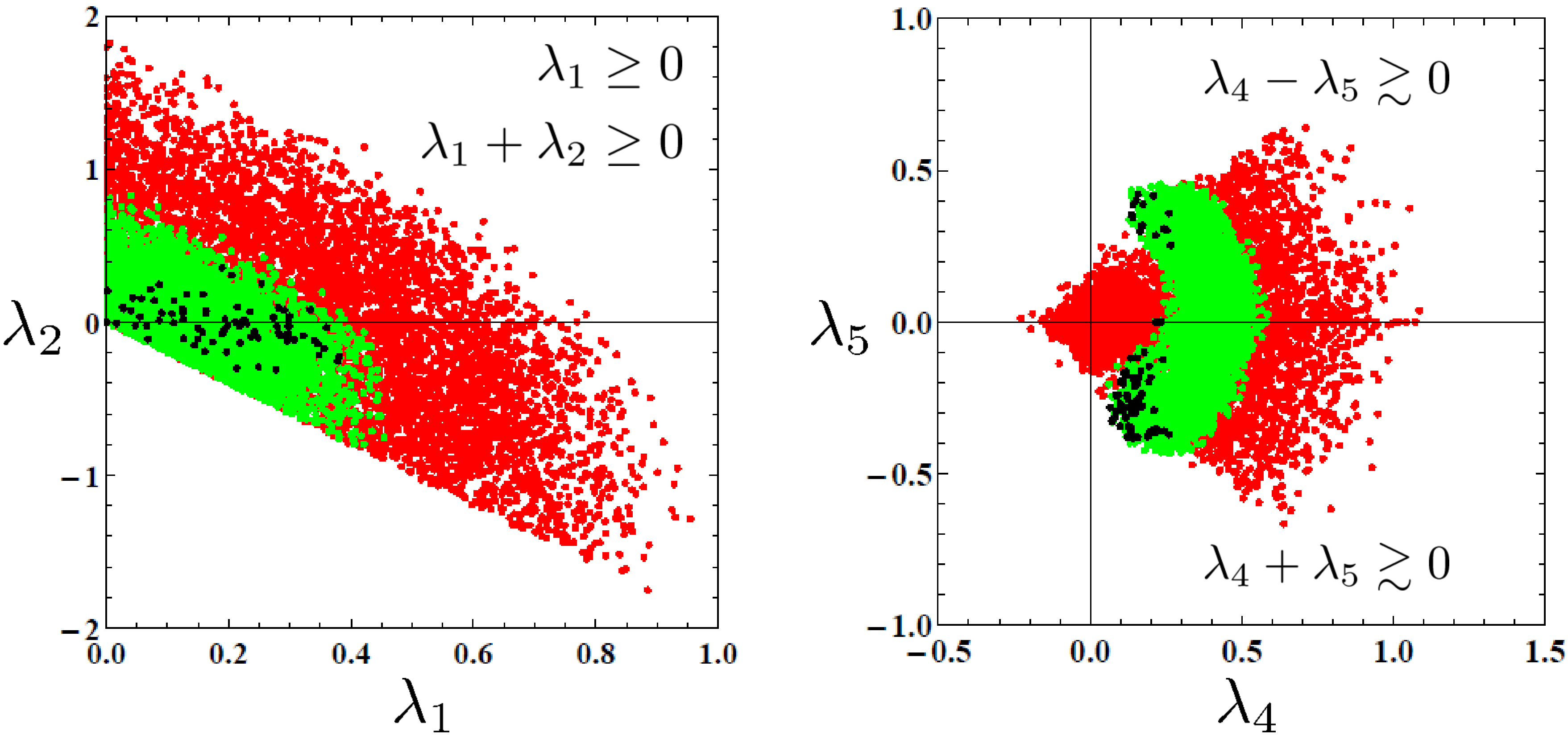
$$v_\Delta M_{H^{\pm\pm}} > 150\,\mathrm{eV\ GeV}$$

['09 A.G.Akeroyd, M.Aoki, H.Sugiyama]
['10 T.Fukuyama, H.Sugiyama, K.Tsumura]

- EW precision data ['12 E.J.Chun, H.M.Lee, P.Sharma]

$$|M_{H^{\pm\pm}} - M_{H^\pm}| < 40\,\mathrm{GeV}$$

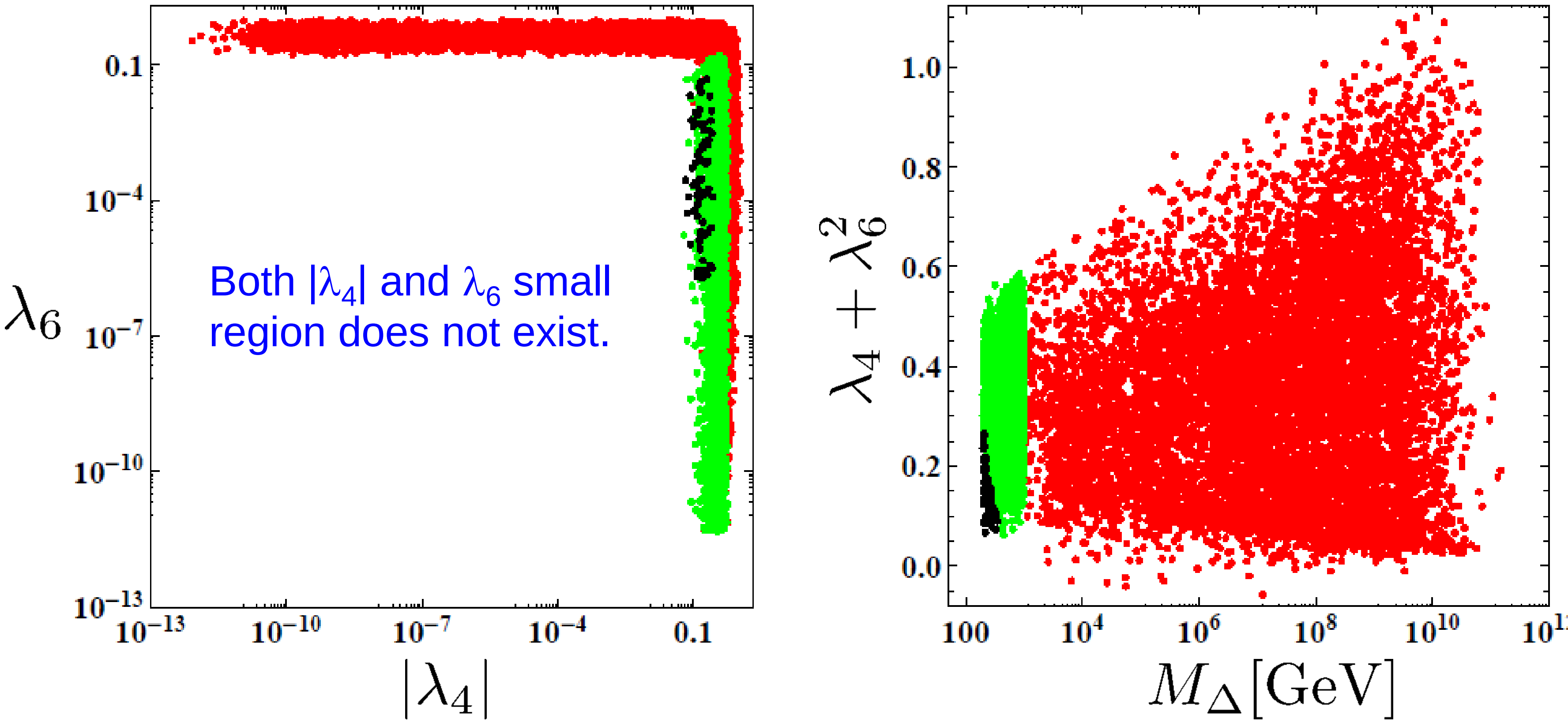
Allowed parameter region



$$\begin{cases} \text{Red} : 200\,\text{GeV} \leq M_\Delta \leq 10^{12}\,\text{GeV} \\ \text{Green} : 200\,\text{GeV} \leq M_\Delta \leq 1\,\text{TeV} \\ \text{Black} : |\delta m_h^2| < M_h^2 \text{ (Naturalness condition)} \end{cases}$$

Large coupling regions are excluded by perturbativity condition.

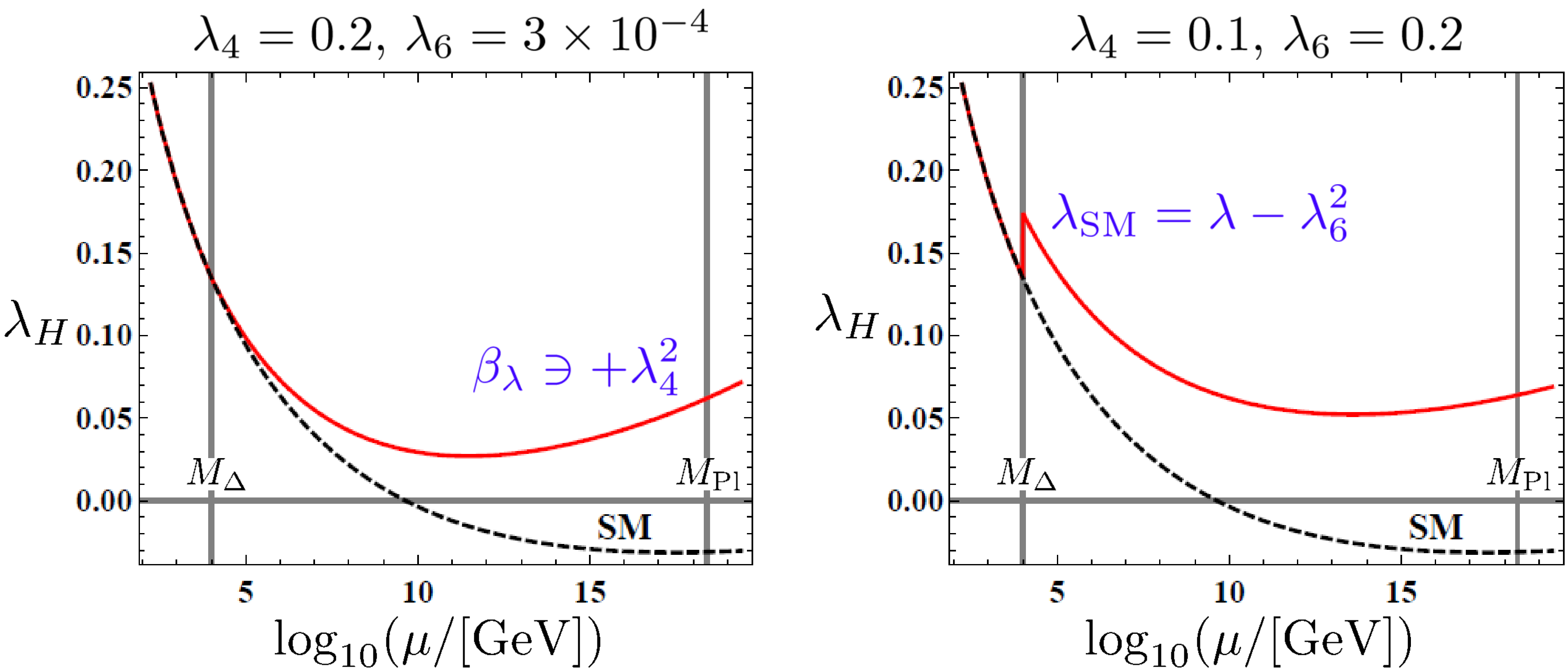
Allowed parameter region



$$\delta m_h^2 \propto (\lambda_4 + \lambda_6^2) M_\Delta^2 \sim M_\Delta^2$$

$$\begin{cases} \text{Red} : 200\,\text{GeV} \le M_\Delta \le 10^{12}\,\text{GeV} \\ \text{Green} : 200\,\text{GeV} \le M_\Delta \le 1\,\text{TeV} \\ \text{Black} : |\delta m_h^2| < M_h^2 \ (\text{Naturalness condition}) \end{cases}$$

Running of $\lambda_H (= \lambda_{\rm SM}$ for $\mu < M_\Delta, \lambda$ for $\mu \geq M_\Delta)$

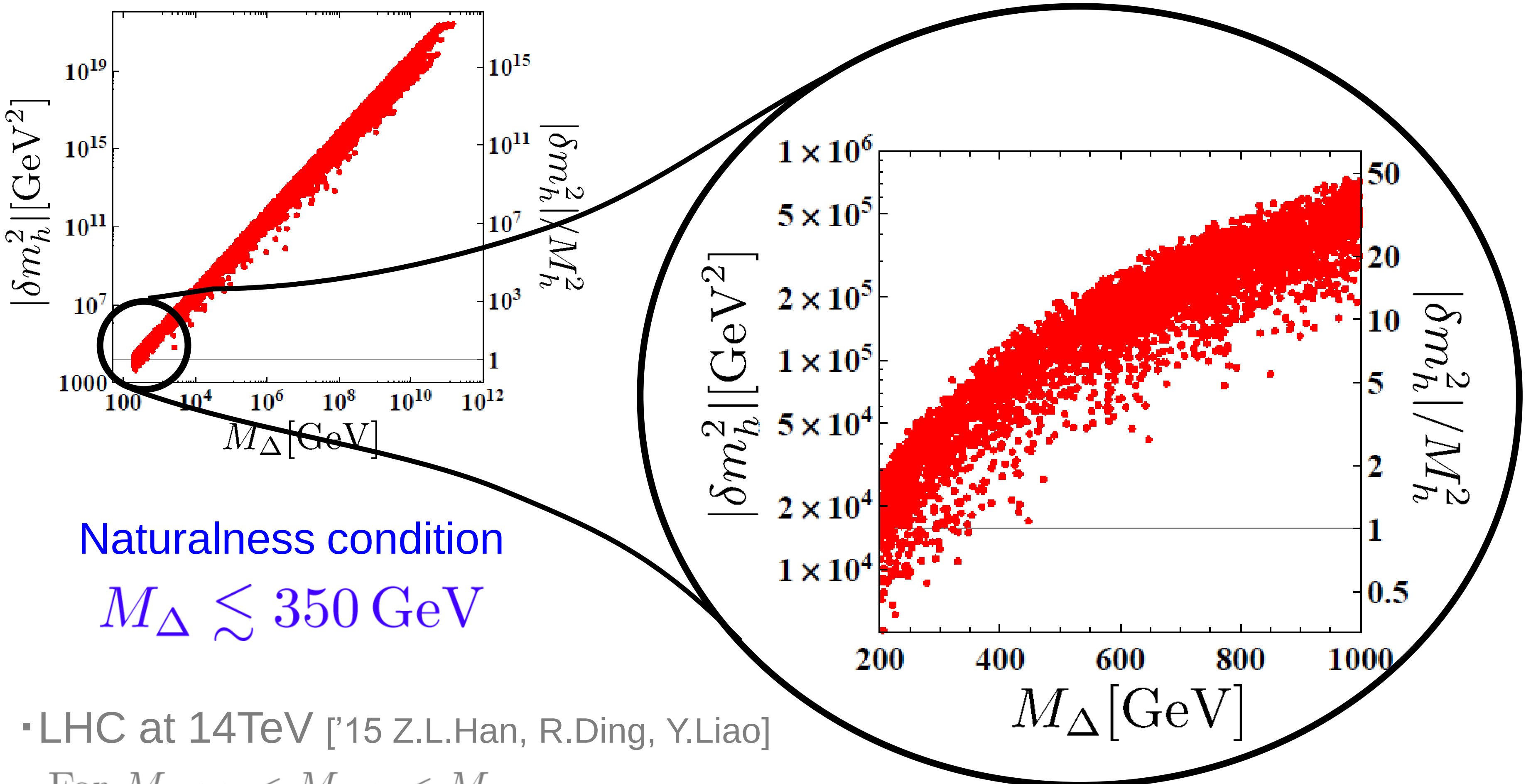


(We have taken $M_\Delta = 10\,{\rm TeV}$ and $\lambda_1 = \lambda_2 = \lambda_5 = 0.1$)

Vacuum stability requires sufficiently large $|\lambda_4|$ and/or λ_6

$\rightarrow$ $\delta m_h^2 \propto (\lambda_4 + \lambda_6^2) M_\Delta^2 \sim M_\Delta^2$ cannot small for large M_Δ

Higgs mass correction



Naturalness condition

$$M_\Delta \lesssim 350\,\mathrm{GeV}$$

- LHC at 14TeV ['15 Z.L.Han, R.Ding, Y.Liao]

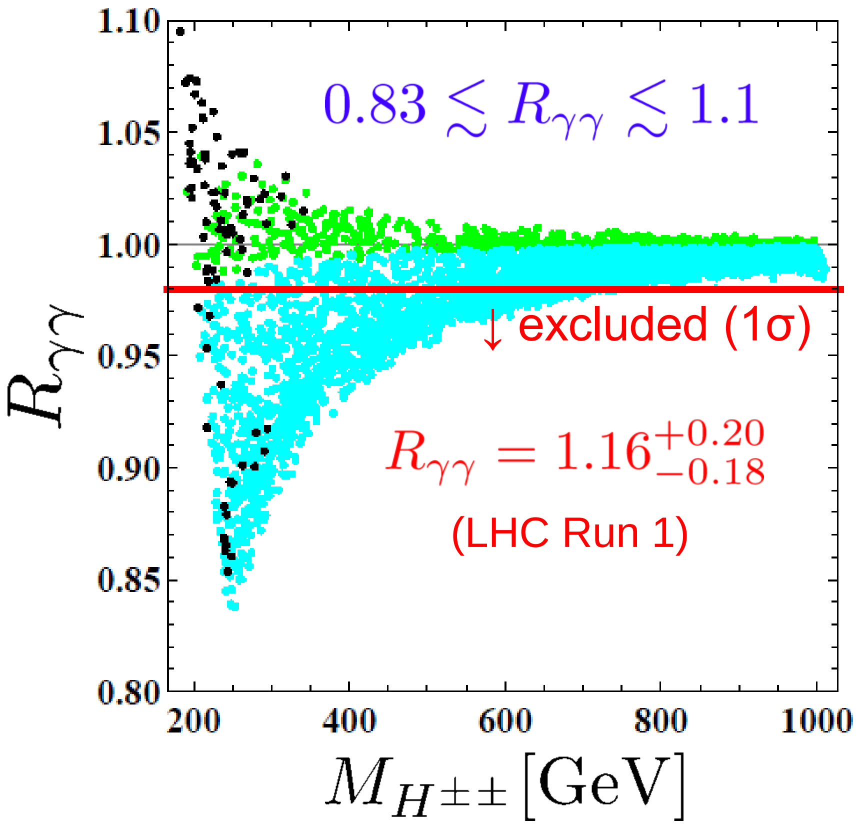
  For $M_{H^{\pm\pm}} < M_{H^\pm} < M_{H^0/A^0}$

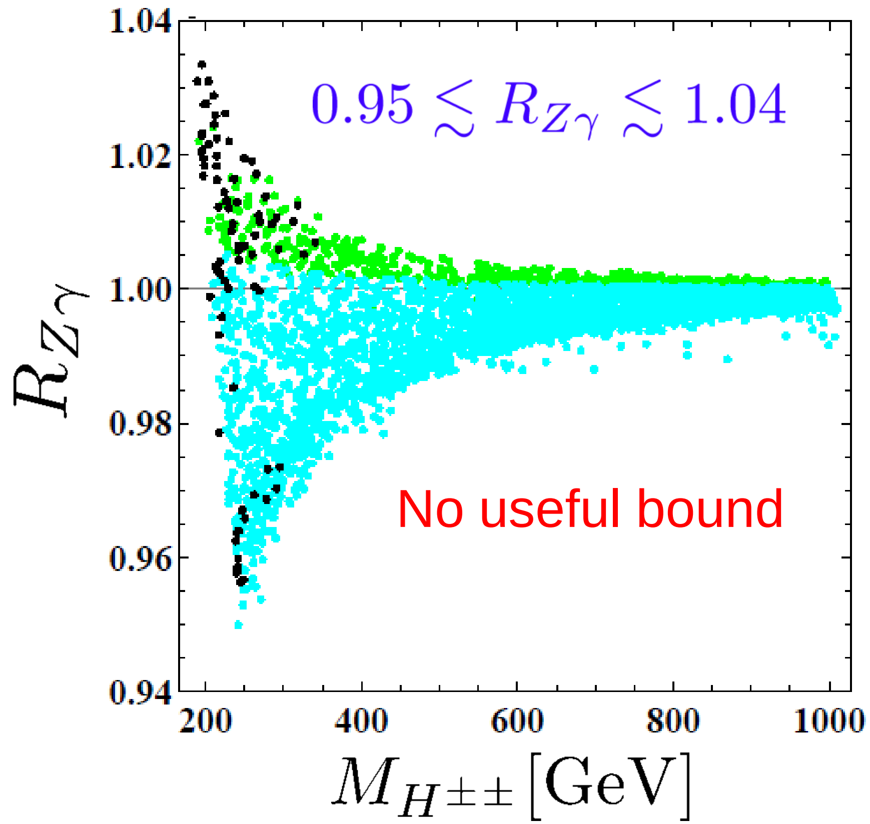
  4ℓ signal with $300\,\mathrm{fb}^{-1}$ can probe up to $M_{H^{\pm\pm}} \sim 600\ (700)\,\mathrm{GeV}$ for NH (IH).

  For $M_{H^{\pm\pm}} > M_{H^\pm} > M_{H^0/A^0}$

  we need an integrated luminosity $\sim 500\,\mathrm{fb}^{-1}$ to reconstruct the triplet scalar.

$h \to \gamma\gamma, Z\gamma$



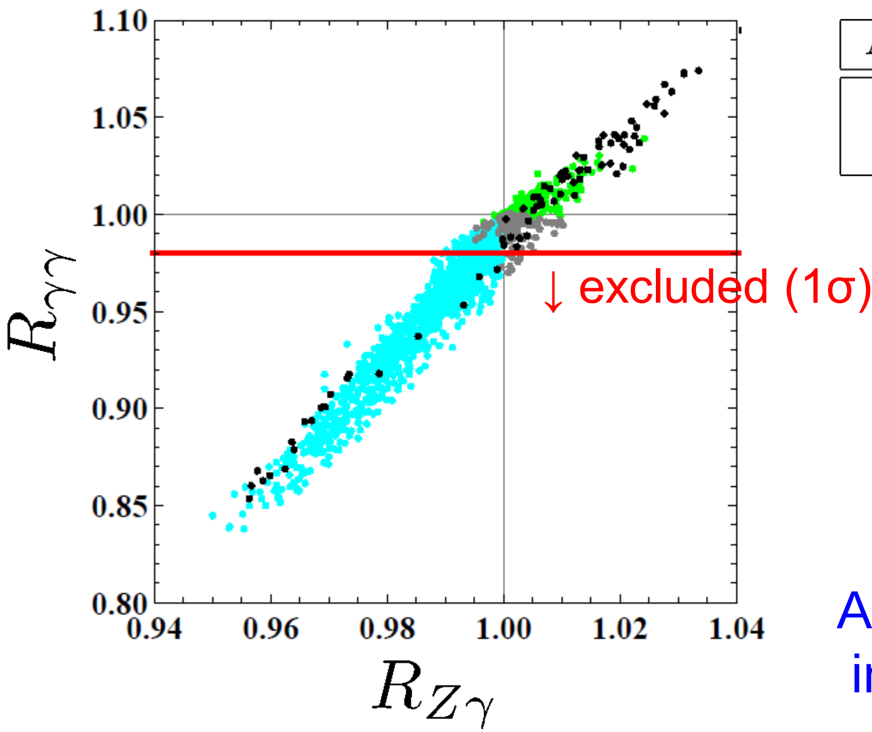


$$\begin{cases} \text{Green} : \lambda_4 + \lambda_5 < 0 \\ \text{Cyan} : \lambda_4 + \lambda_5 > 0 \\ \text{Black} : |\delta m_h^2| < M_h^2 \end{cases}$$

Amplitude	Fermions	W-boson (dominant)	$H^{\pm}$	$H^{\pm\pm}$ (sub-dominant)
$A^{\gamma\gamma}$	+	−	λ_4	$\lambda_4 + \lambda_5$
$A^{Z\gamma}$	−	+	λ_4	$-(\lambda_4 + \lambda_5)$

Both R is enhanced when $\lambda_4 + \lambda_5 < 0$.

$h \to \gamma\gamma, Z\gamma$



Amplitude	Fermions	W-boson	$H^{\pm}$	$H^{\pm\pm}$
$A^{\gamma\gamma}$	$+$	$-$	λ_4	$\lambda_4+\lambda_5$
$A^{Z\gamma}$	$-$	$+$	λ_4	$-(\lambda_4+\lambda_5)$

$$\begin{cases} \text{Green}: \lambda_4+\lambda_5 < -0.05 \\ \text{Gray}: -0.05 < \lambda_4+\lambda_5 < 0.05 \\ \text{Cyan}: \lambda_4+\lambda_5 > 0.05 \\ \text{Black}: |\delta m_h^2| < M_h^2 \end{cases}$$

Anti-correlation can be seen in gray region, in which $M_{H^{\pm}}$ is sub-dominant.

- LHC at 14TeV ['14 ATLAS]

To prove our result,
we require an integrated luminosity $\sim 3000\,\mathrm{fb}^{-1}$ at LHC 14 TeV.
Then, the expected measured signal for $h \to Z\gamma \to \ell\ell\gamma$ is

$$R_{Z\gamma(\to\ell\ell\gamma)} = 1.00^{+0.25}_{-0.26}(\text{stat.})^{+0.17}_{-0.15}(\text{sys.}).$$

Summary

- Hierarchy problem can be avoided when $gM^2/\Lambda_{\rm EW}^2 \lesssim 1$
- We investigate minimal type-II seesaw model
  - Vacuum stability and Perturbativity conditions
  - Naturalness condition
- From Naturalness condition, we have obtained
  - $M_\Delta \lesssim 350\,{\rm GeV}$
  - $0.83 \lesssim R_{\gamma\gamma} \lesssim 1.1$ and $0.95 \lesssim R_{Z\gamma} \lesssim 1.04$
- Our results can (may) be tested by LHC Run 2

Backup

Mass spectrum

There are seven physical massive eigenstates:

$$
\begin{aligned}
M_{H^{\pm\pm}}^2 &= M_\Delta^2 + \frac{1}{2}(\lambda_4+\lambda_5)v^2 + \frac{1}{2}(\lambda_1+\lambda_2)v_\Delta^2, \\
M_{H^\pm}^2 &= \left(M_\Delta^2 + \frac{1}{2}\lambda_4 v^2 + \frac{1}{2}\lambda_1 v_\Delta^2\right)\left(1+\frac{2v_\Delta^2}{v^2}\right), \\
M_{A^0}^2 &= \left(M_\Delta^2 + \frac{1}{2}(\lambda_4-\lambda_5)v^2 + \frac{1}{2}\lambda_1 v_\Delta^2\right)\left(1+\frac{4v_\Delta^2}{v^2}\right), \\
M_h^2 &= \frac{1}{2}\left(A+C-\sqrt{(A-C)^2+4B^2}\right), \\
M_{H^0}^2 &= \frac{1}{2}\left(A+C+\sqrt{(A-C)^2+4B^2}\right)
\end{aligned}
$$

$$
\text{with} \qquad A=\lambda v^2, \quad B=-\frac{2v_\Delta}{v}\left(M_\Delta^2+\frac{1}{2}\lambda_1 v_\Delta^2\right),
$$

$$
C = M_\Delta^2 + \frac{1}{2}(\lambda_4-\lambda_5)v^2 + \frac{3}{2}\lambda_1 v_\Delta^2 .
$$

h → γ γ, Z γ

$$\Gamma(h\to\gamma\gamma) = \frac{\alpha_{\rm em}^2 G_F M_h^3}{128\sqrt{2}\pi^3}\left|\sum_f N_c Q_f^2 g_{hf\bar f} A_{1/2}^{\gamma\gamma}(\tau_h^f) + g_{hW^+W^-}A_1^{\gamma\gamma}(\tau_h^W)\right.$$
$$\left. +\tilde g_{hH^\pm H^\mp}A_0^{\gamma\gamma}(\tau_h^{H^\pm}) + 4\tilde g_{hH^{\pm\pm}H^{\mp\mp}}A_0^{\gamma\gamma}(\tau_h^{H^{\pm\pm}})\right|^2$$

$$\Gamma(h\to Z\gamma) = \frac{\alpha_{\rm em}^2 M_h^3}{128\pi^3 v^2}\left(1-\frac{M_Z^2}{M_h^2}\right)^3\left|\frac{1}{s_W c_W}\sum_f N_c Q_f(2I_3^f - 4Q_f s_W^2)A_{1/2}^{Z\gamma}(\tau_h^f,\tau_Z^f)\right.$$
$$+\cot\theta_W g_{hW^+W^-}A_1^{Z\gamma}(\tau_h^W,\tau_Z^W) - 2g_{ZH^\pm H^\mp}\tilde g_{hH^\pm H^\mp}A_0^{Z\gamma}(\tau_h^{H^\pm},\tau_Z^{H^\pm})$$
$$\left.-4g_{ZH^{\pm\pm}H^{\mp\mp}}\tilde g_{hH^{\pm\pm}H^{\mp\mp}}A_0^{Z\gamma}(\tau_h^{H^{\pm\pm}},\tau_Z^{H^{\pm\pm}})\right|^2$$

$h \to \gamma\gamma, Z\gamma$

$$\text{with } \tau_h^i = 4M_i^2/M_h^2, \;\; \tau_Z^i = 4M_i^2/M_Z^2 \;\; (i = f, W, H^\pm, H^{\pm\pm})$$

$$\begin{aligned}
A_0^{\gamma\gamma}(x) &= -x^2[x^{-1} - f(x^{-1})]\,, \\
A_{1/2}^{\gamma\gamma}(x) &= 2x^2[x^{-1} + (x^{-1}-1)f(x^{-1})]\,, \\
A_1^{\gamma\gamma}(x) &= -x^2[2x^{-2} + 3x^{-1} + 3(2x^{-1}-1)f(x^{-1})]\,, \\
A_0^{Z\gamma}(x,y) &= I_1(x,y)\,, \\
A_{1/2}^{Z\gamma}(x,y) &= I_1(x,y) - I_2(x,y)\,, \\
A_1^{Z\gamma}(x,y) &= 4(3-\tan^2\theta_W)I_2(x,y) + [(1+2x^{-1})\tan^2\theta_W - (5+2x^{-1})]I_1(x,y)
\end{aligned}$$

$$f(x) = (\sin^{-1}\sqrt{x})^2\,, \qquad g(x) = \sqrt{x^{-1}-1}(\sin^{-1}\sqrt{x})$$

$$\begin{aligned}
I_1(x,y) &= \frac{xy}{2(x-y)} + \frac{x^2y^2}{2(x-y)^2}[f(x^{-1}) - f(y^{-1})] + \frac{x^2y}{(x-y)^2}[g(x^{-1}) - g(y^{-1})]\,, \\
I_2(x,y) &= -\frac{xy}{2(x-y)}[f(x^{-1}) - f(y^{-1})]
\end{aligned}$$

$h \to \gamma\gamma, Z\gamma$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta' & \sin\beta' \\ -\sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \delta^\pm \end{pmatrix},$$

$$\begin{pmatrix} h \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi \\ \delta \end{pmatrix},$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi \\ \eta \end{pmatrix}$$

$$g_{hf\bar{f}} = \frac{\cos\alpha}{\cos\beta'}, \qquad g_{hW^+W^-} = \cos\alpha + 2\frac{v_\Delta}{v}\sin\alpha$$

$$g_{ZH^+H^-} = -\tan\theta_W, \qquad g_{ZH^{++}H^{--}} = 2\cot 2\theta_W$$

$$\tilde{g}_{hH^+H^-} = \frac{M_W}{gM_{H^\pm}^2} g_{hH^+H^-}, \qquad \tilde{g}_{hH^{++}H^{--}} = \frac{M_W}{gM_{H^{\pm\pm}}^2} g_{hH^{++}H^{--}}$$

$$\begin{aligned} g_{hH^+H^-} &= \left[\left(\lambda_1\cos^2\beta' + (\lambda_4+\lambda_5)\sin^2\beta'\right)v_\Delta + \sqrt{2}\lambda_5\cos\beta'\sin\beta' v\right]\sin\alpha + \\ &\quad \left[\left(\lambda\sin^2\beta' + \lambda_4\cos^2\beta'\right)v + \sqrt{2}\cos\beta'\sin\beta'\left(\frac{2M_\Delta^2}{v^2}+\lambda_4\right)v_\Delta\right]\cos\alpha, \\ g_{hH^{++}H^{--}} &= (\lambda_1+\lambda_2)v_\Delta\sin\alpha + (\lambda_4+\lambda_5)v\cos\alpha \end{aligned}$$

$$g_{hH^+H^-} \simeq \lambda_4 v, \qquad g_{hH^{++}H^{--}} \simeq (\lambda_4+\lambda_5)v$$