Dark matter and electroweak scalegenesis from a bilinear scalar condensate

Masatoshi Yamada
(Kanazawa University)

Collaborator: Jisuke Kubo (Kanazawa University)

Based on

J, Kubo and **M. Y.**, arXiv:1505.05971 (to be published in PRD)

J, Kubo and **M. Y.**, PTEP **2015** 093B01 (arXiv:1506.06460)

Workshop of elementary particle physics in Matsue @Shimane Univ.

Introduction

- The SM is complete.
- Still unsolved problems.
- In this talk,
  - Fine-tuning problem (criticality problem)
  - The origin of electroweak symmetry breaking
  - Dark matter
- Suggest a model based on the <u>classical scale invariance.</u>

Classical scale invariance and Scalegenesis

- The Higgs mass is prohibited.
  - The SM becomes scaleless.
- How to generate a scale?
  - Coleman-Weinberg mechanism
    - Perturbative
    - The mass term is not generated.

$$V_{\mathrm{eff}}(v_h) = \frac{\lambda_H}{4} v_h^4 + \sum_{\alpha} \frac{N_\alpha M_\alpha^4}{64\pi^2} \left(\log \left(\frac{M_\alpha^2}{\mu^2} \right) - C_\alpha \right)$$

  - Strong dynamics
    - Non-perturbative
    - The dynamical mass term is generated.

$$M \sim \langle \bar{\psi} \psi \rangle \propto \Lambda_{\mathrm{QCD}}$$

Contents

1. The model
2. Dark matter candidates
3. 1st order phase transition of electroweak symmetry (at finite temperature)

Contents

1. The model
2. Dark matter candidates
3. 1st order phase transition of electroweak symmetry (at finite temperature)

Model

J, Kubo and **M. Y.**, arXiv:1505.05971

- Strongly interacting Hidden sector
  - $SU(N_c)\times U(N_f)$ invariant + classically scale invariant

$$\mathcal{L}_{\mathrm{HQCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + ([D_\mu S_i]^\dagger D^\mu S_i) - \hat{\lambda}_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \hat{\lambda}'_S (S_i^\dagger S_j)(S_j^\dagger S_i) + \hat{\lambda}_{HS}(S_i^\dagger S_i)H^\dagger H$$

$$\langle S^\dagger S \rangle$$

$$m_{\mathrm{H}}^2 = -\lambda_{HS}\langle S^\dagger S \rangle$$

Dynamical Scale Symmetry Breaking

$$\mathcal{L}_{\mathrm{SM}}\Big|_{m_{\mathrm{H}}\to 0} \quad -\lambda_{HS}(S^\dagger S)H^\dagger H \quad \mathcal{L}_{\mathrm{HQCD}}$$

Effective theory

- Low energy effective Lagrangian

$$\mathcal{L}_{\rm eff} = ([\partial_\mu S_i]^\dagger \partial^\mu S_i) + \lambda_{HS}(S_i^\dagger S_i) H^\dagger H$$

$$-\lambda_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \lambda'_S (S_i^\dagger S_j)(S_j^\dagger S_i)$$

  - Assume that DSSB is dominant.
  - Attempt to describe the genesis of scale *a la* Coleman-Weinberg.
  - Scale invariant Lagrangian.
  - Renormalizable
  - λ_S, λ'_S and λ_{HS}: effective coupling constants.
    - which contain the quantum effects of hidden gluon.

Comparison

$$\mathcal{L}_{\mathrm{HQCD}}$$

Scale invariant

$$\mathcal{L}_{\mathrm{QCD}}$$

Chiral invariant

Effective model

$$V_{\mathrm{eff}} = \lambda_S (S_i^\dagger S_i)(S_j^\dagger S_j) + \lambda'_S (S_i^\dagger S_j)(S_j^\dagger S_i) - \lambda_{HS}(H^\dagger H)(S^\dagger S)$$

$$V_{\mathrm{eff}} = G[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma^5 \tau^\alpha \psi)^2]$$

Order parameter

$$\langle S^\dagger S \rangle \neq 0$$

$$\langle \bar{\psi}\psi \rangle \neq 0$$

meson

σ ϕ^α

σ π^α

Effective potential

- The mean-field approximated effective potential
  - Integrate out χ (Gauss integral) $\quad S_i \to S_i + \chi_i$

$$V_{\mathrm{MFA}} = M^2 (S_i^\dagger S_i) + \lambda_{\mathrm{H}} (H^\dagger H)^2 - N_{\mathrm{f}} (N_{\mathrm{f}} \lambda_S + \lambda_S') f^2 + \frac{N_{\mathrm{f}} N_{\mathrm{c}}}{32\pi^2} M^4 \ln \frac{M^2}{\Lambda_H^2}$$

$\overline{\mathrm{MS}}$ scheme

$$M^2 = 2(N_{\mathrm{f}} \lambda_S + \lambda_S') f - \lambda_{HS} H^\dagger H$$

Tr log χ

- Solving the gap equations

$$\langle S \rangle = 0, \quad \langle f \rangle \neq 0, \quad \langle H \rangle \neq 0$$

Solutions

- The vacuum of Higgs

$$\langle h \rangle = \frac{N_{\rm f}\lambda_{HS}}{G}\Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right)$$

- The scalar condensate

$$\langle S^\dagger S \rangle = \langle f \rangle = \frac{2\lambda_H}{G}\Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right)$$

- Constituent scalar mass

$$M^2 = \Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right)$$

$$G = 4N_{\rm f}\lambda_H\lambda_S - N_{\rm f}\lambda_{HS}^2 + 4\lambda_H\lambda_S'$$

Input & free parameters

- Input
  - Higgs mass $m_{\rm H} = 126$ GeV
  - EW vacuum $\langle h \rangle = 246$ GeV
  - DM relic abundance $\Omega \hat{h}^2 \sim 0.12$
- 7 free parameters.

$$\lambda_S \qquad \lambda'_S \qquad \lambda_{HS} \qquad \lambda_H$$

$$N_{\rm f} \qquad N_{\rm c} \qquad \Lambda_H$$

Contents

1. The model
2. Dark matter candidates
3. 1^{st} order phase transition of electroweak symmetry (at finite temperature)

Dark matter candidate

Dark matter candidate is ϕ^{α}

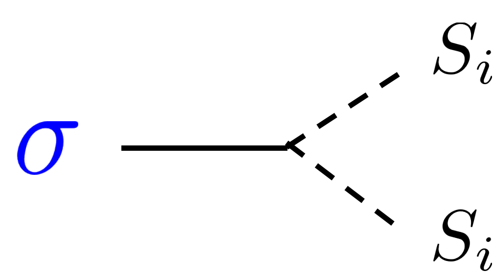
- The excitation fields from the vacuum $< S^{\dagger} S >$
  - Assume the unbroken $\mathrm{U}(N_{\mathrm{f}})$ flavor symmetry:

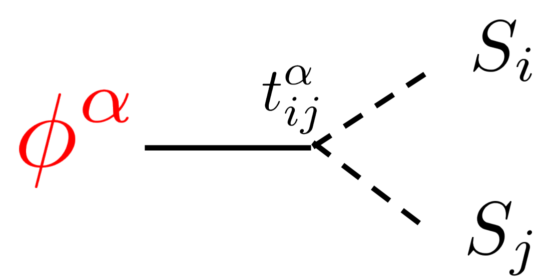
$$\langle \Omega | (S_i^{\dagger} S_j) | \Omega \rangle = f_0 \delta_{ij} + \delta_{ij} Z_{\sigma}^{\frac{1}{2}} \sigma + t_{ji}^{\alpha} Z_{\phi}^{\frac{1}{2}} \phi^{\alpha}$$

c.f. $\langle \Omega | \bar{\psi}_i \psi_j | \Omega \rangle = f_0 \delta_{ij} + \delta_{ij} Z_{\sigma}^{\frac{1}{2}} \sigma + t_{ji}^{\alpha} Z_{\pi}^{\frac{1}{2}} \pi^{\alpha}$

- Mean-field Lagrangian (before integrating S)

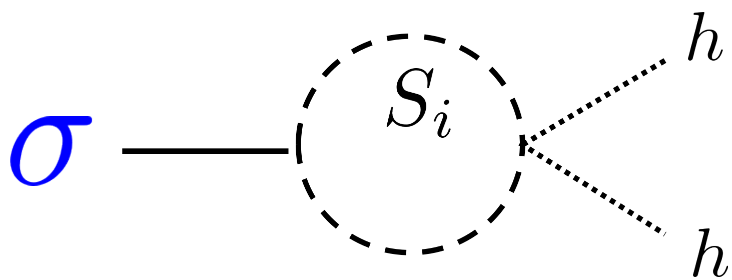
$\mathcal{L}'_{\mathrm{MFA}} \supset$

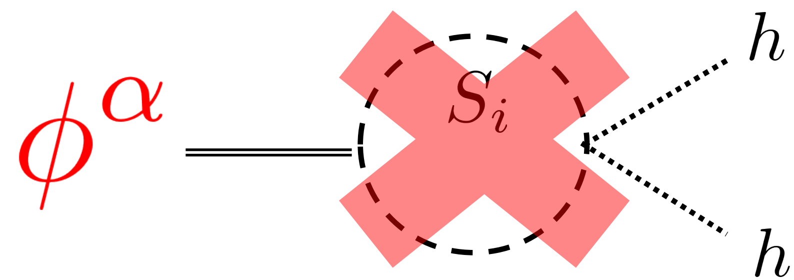




Dark matter candidate is ϕ^{α}

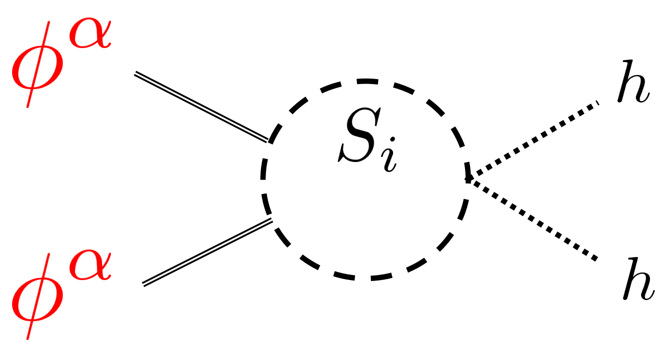
- Decay into Higgs through S loop





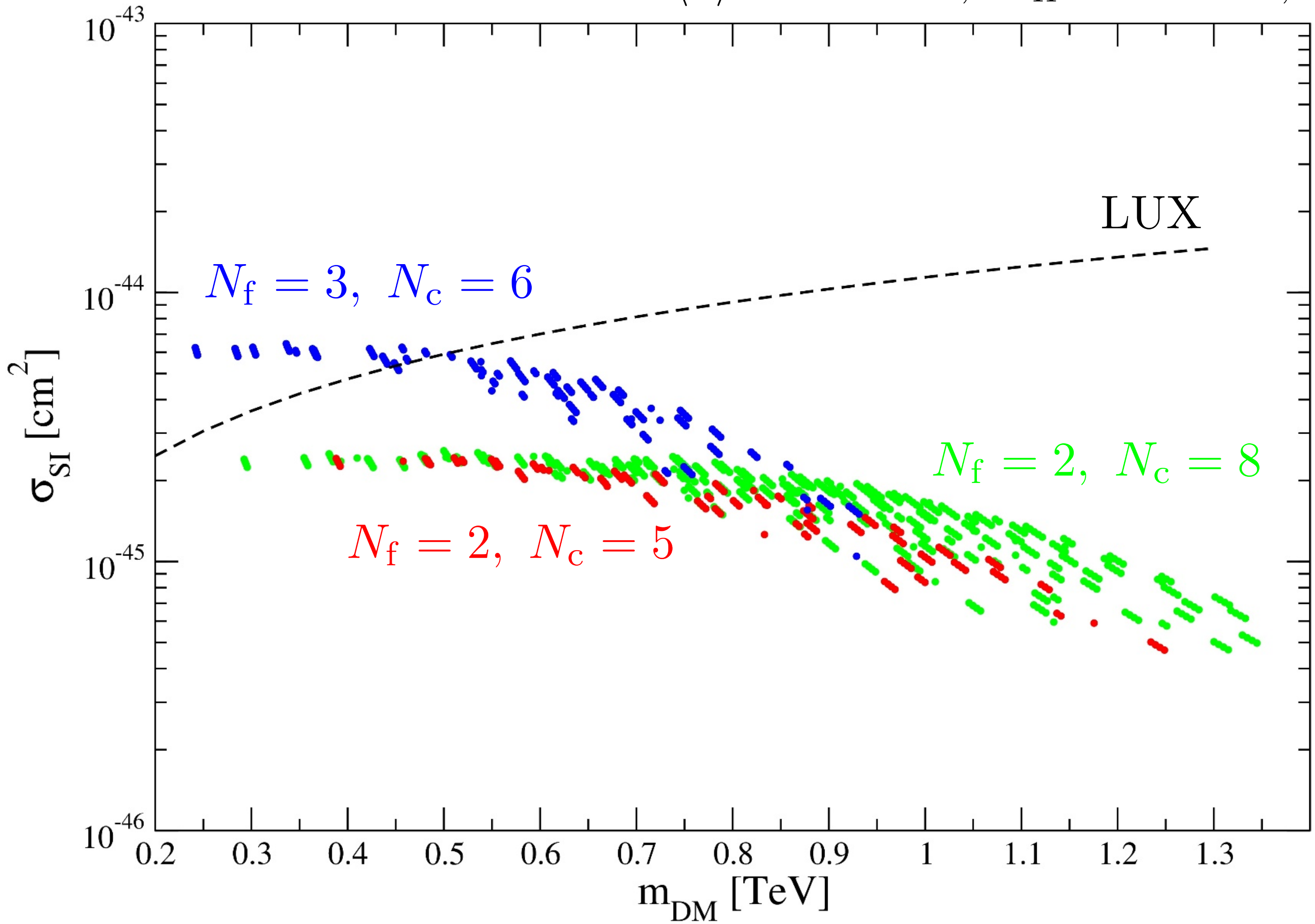
Forbidden by flavor symmetry

- Coannihilation



σ_{SI} vs. m_{DM}

$\langle h \rangle = 246\ \mathrm{GeV},\ m_{\mathrm{H}} = 126\,\mathrm{GeV},\ \Omega\hat{h}^2 \sim 0.12$



Contents

1. The model
2. Dark matter candidates
3. 1st order phase transition of electroweak symmetry (at finite temperature)

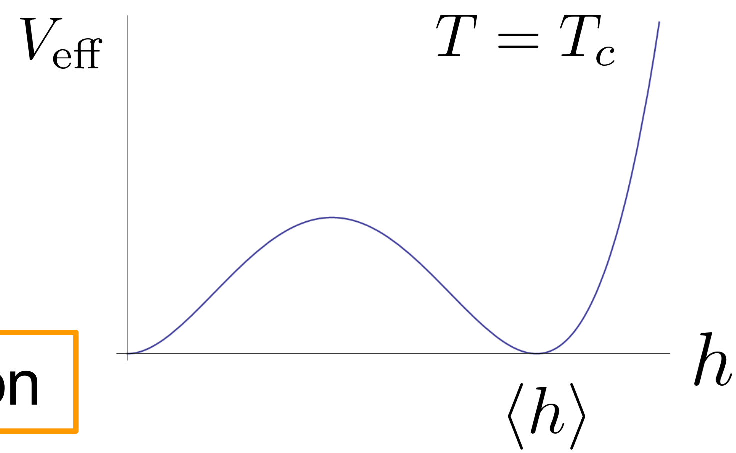
Electroweak 1st order phase transition

EW Baryogenesis scenario

- Sakharov conditions
  1. Baryon number violation
  2. C-symmetry and CP-symmetry violation
  3. Interactions out of thermal equilibrium.

- Electroweak strong first-order phase transition

$$\frac{\langle h \rangle}{T_c} \gtrsim 1$$



The SM cannot satisfy this condition

Phase transition

- V_{eff} at zero temperature

$$V_{\text{eff}}(f, h; T = 0) = \quad V_{\text{MFA}}(f, h) + V_{\text{CW}}(h)$$

$$\Rightarrow \langle h \rangle = 246 \text{ GeV} \quad \langle f \rangle \neq 0$$

$$m_{\text{H}} = 126 \text{ GeV}$$

- V_{eff} at critical temperature T_{c}^{EW}(EWPT)

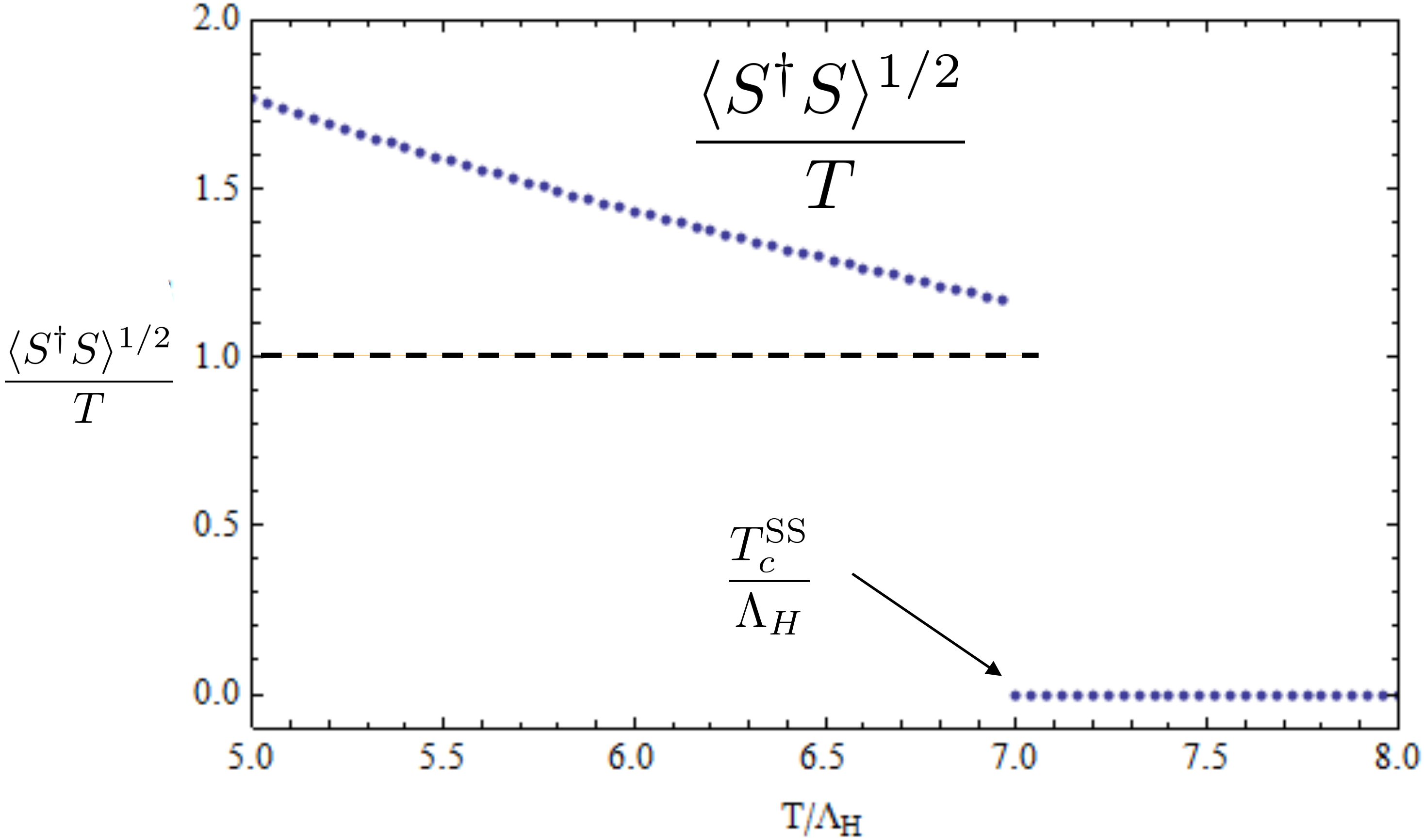
$$V_{\text{eff}}(f, h; T = T_c^{\text{EW}}) \Rightarrow \langle h \rangle = 0$$

- V_{eff} at critical temperature T_{c}^{SS} (SSPT)

$$V_{\text{eff}}(f, h; T = T_c^{\text{SS}}) \Rightarrow \langle f \rangle = \langle S^\dagger S \rangle = 0$$

Scale transition is strong 1st order.

J, Kubo and **M. Y.**, PTEP **2015** 093B01 (arXiv:1506.06460)

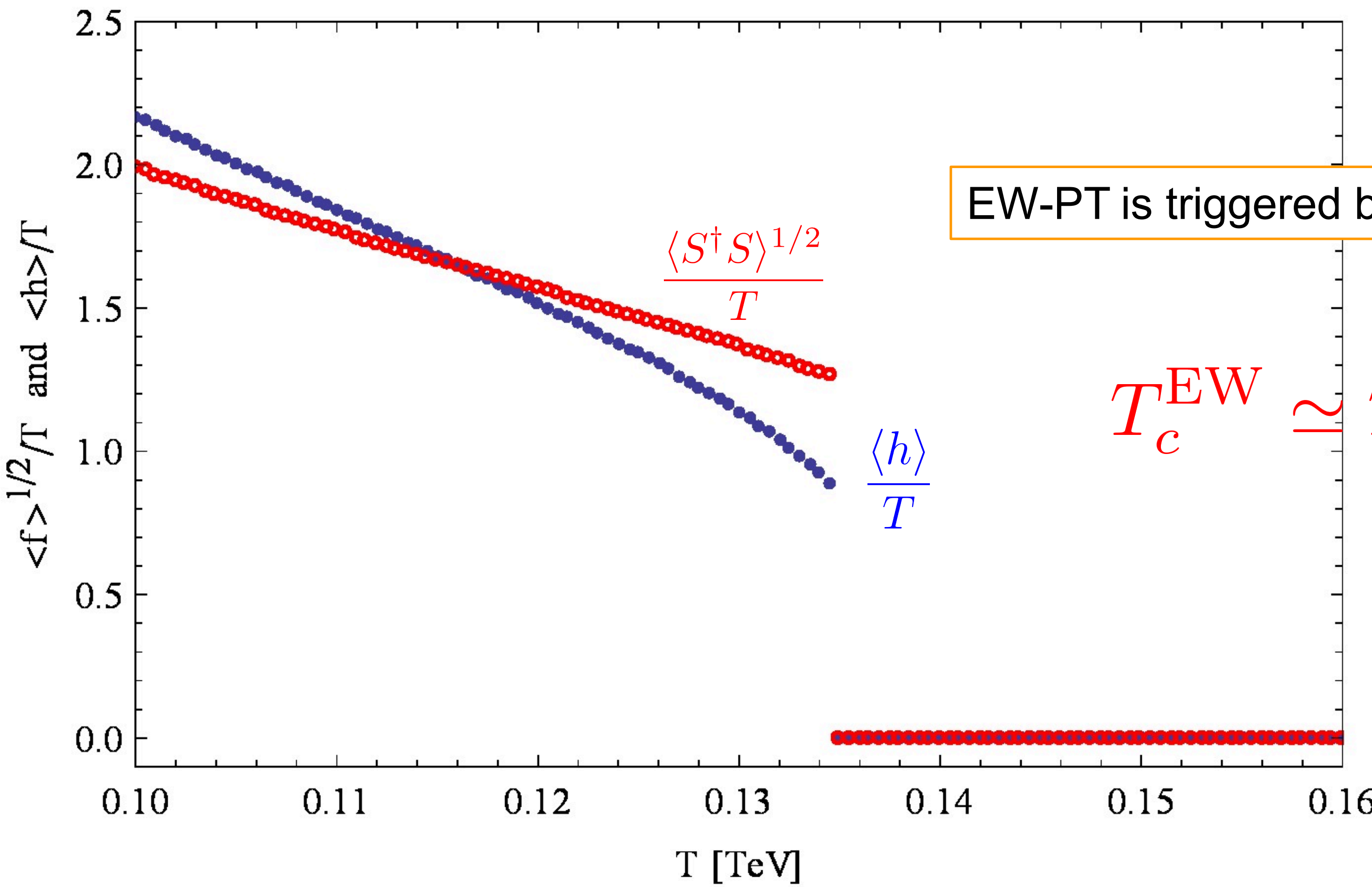


Without dark matter case: $N_f = 1$

EW phase transition becomes strong 1st order

$\langle h \rangle = 246\ \text{GeV},\ m_H = 126\,\text{GeV}$

$N_c = 6$



EW-PT is triggered by SS-PT

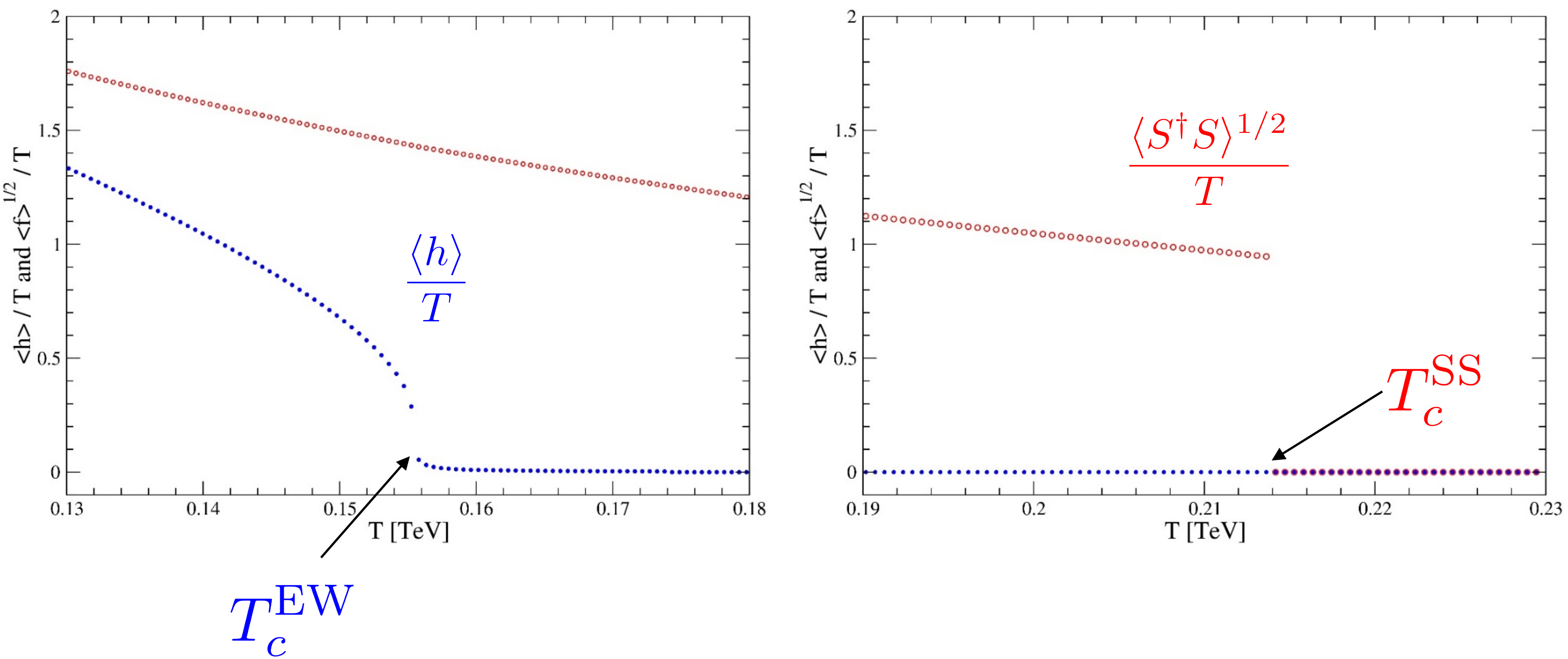
$T_c^{EW} \simeq T_c^{SS}$

With dark matter case: $N_{\mathrm{f}} = 2$

EW phase transition becomes weak 1st order

$$\langle h \rangle = 246\ \mathrm{GeV},\ m_{\mathrm{H}} = 126\,\mathrm{GeV},\ \Omega\hat{h}^2 \sim 0.12$$

$$N_{\mathrm{c}} = 6$$



Difference between two cases

- The Higgs portal is important

$$-\lambda_{HS}(S^\dagger S)H^\dagger H$$

$\uparrow \lambda_{\mathrm{HS}}$ EW-PT is triggered. $\Omega\hat{h}^2 \sim 0.12$

$\downarrow \lambda_{\mathrm{HS}}$ Not enough to trigger $\Omega\hat{h}^2 \sim 0.12$

- Need more precisely analysis

Summary

- We suggested a new model based on classically scale invariance.
  - Strongly interacting hidden sector with the scalar field
  - Explain the mechanism of generation of "scale"
  - Dynamical Scale Symmetry Breaking $< S^{\dagger} S > \neq 0$
  - The EW symmetry breaking $< h > \neq 0$

"Scalegenesis" is realized!

Summary

- We suggested a new model based on classically scale invariance.
  - Strongly interacting hidden sector with the scalar field
  - Explain the mechanism of generation of "scale"
  - Dynamical Scale Symmetry Breaking $< S^{\dagger}S > \neq 0$
  - The EW symmetry breaking $< h > \neq 0$

"Scalegenesis" is realized!

- Dark matter candidate exists.
- The EW 1st order phase transition

Prospects

- More precise analysis is needed.
  - Lattice simulation
- C and CP violation

Appendix

Hierarchy problem

- Nothing between Λ_{EW} and Λ_{pl}?
  - $\Lambda_{\mathrm{EW}} \sim \mathcal{O}(10^2)\ \mathrm{GeV} \Leftrightarrow \Lambda_{\mathrm{pl}} \sim \mathcal{O}(10^{19})\ \mathrm{GeV}$
- Fine-tuning problem

$$m_R^2 = m_0^2 - \left(\frac{\lambda}{16\pi^2} + \cdots \right) \Lambda_{\mathrm{pl}}^2$$

$$(10^2\ \mathrm{GeV})^2 = (10^{19}\ \mathrm{GeV})^2 - (10^{19}\ \mathrm{GeV})^2$$

- Fermion and gauge field have not the problem.
  - Gauge symmetry: $\cancel{m_0^2 A_\mu A^\mu}$ $\quad m_Z^2 \propto \langle h \rangle^2 \sim \Lambda_{\mathrm{EW}}^2$
  - Chiral symmetry: $\cancel{m_0 \bar{\psi} \psi}$ $\quad m_q^2 \propto \langle \bar{\psi} \psi \rangle \sim \Lambda_{\mathrm{QCD}}^2$

Argument by Bardeen

W.A. Bardeen, On naturalness in the standard model, **FERMILAB-CONF-95-391** (1995).

- The quadratic divergences are spurious.
  - Λ always is subtracted by renormalization.
  - The dimensional regularization automatically subtracts the quadratic divergence.

- Only logarithmic terms related to the scale anomaly survive in the perturbation.

⇒ The non-zero beta function $\beta \neq 0$

Argument by Bardeen

W.A. Bardeen, On naturalness in the standard model, **FERMILAB-CONF-95-391** (1995).

- The RG equation of Higgs mass

$$\frac{dm^2}{d\log\mu} = \frac{m^2}{16\pi^2}\left(12\lambda + 6y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_1^2\right)$$

  - If $m(\Lambda_{\mathrm{pl}}) = 0$, the mass dose not run.

- If the Higgs field is coupled to a new particle with mass M,

$$m_R^2 = m_0^2 + \frac{\lambda'}{16\pi^2}M^2\log\left(\frac{\mu^2}{M^2}\right) + \cdots$$

  - If $M \sim \mathcal{O}(\mathrm{TeV})$, fine-tuning is not needed.
    - Even if so, the origin of m_0 with TeV order is unknow.
  - If $M \gg \mathrm{TeV}$, fine-tuning problem appears.

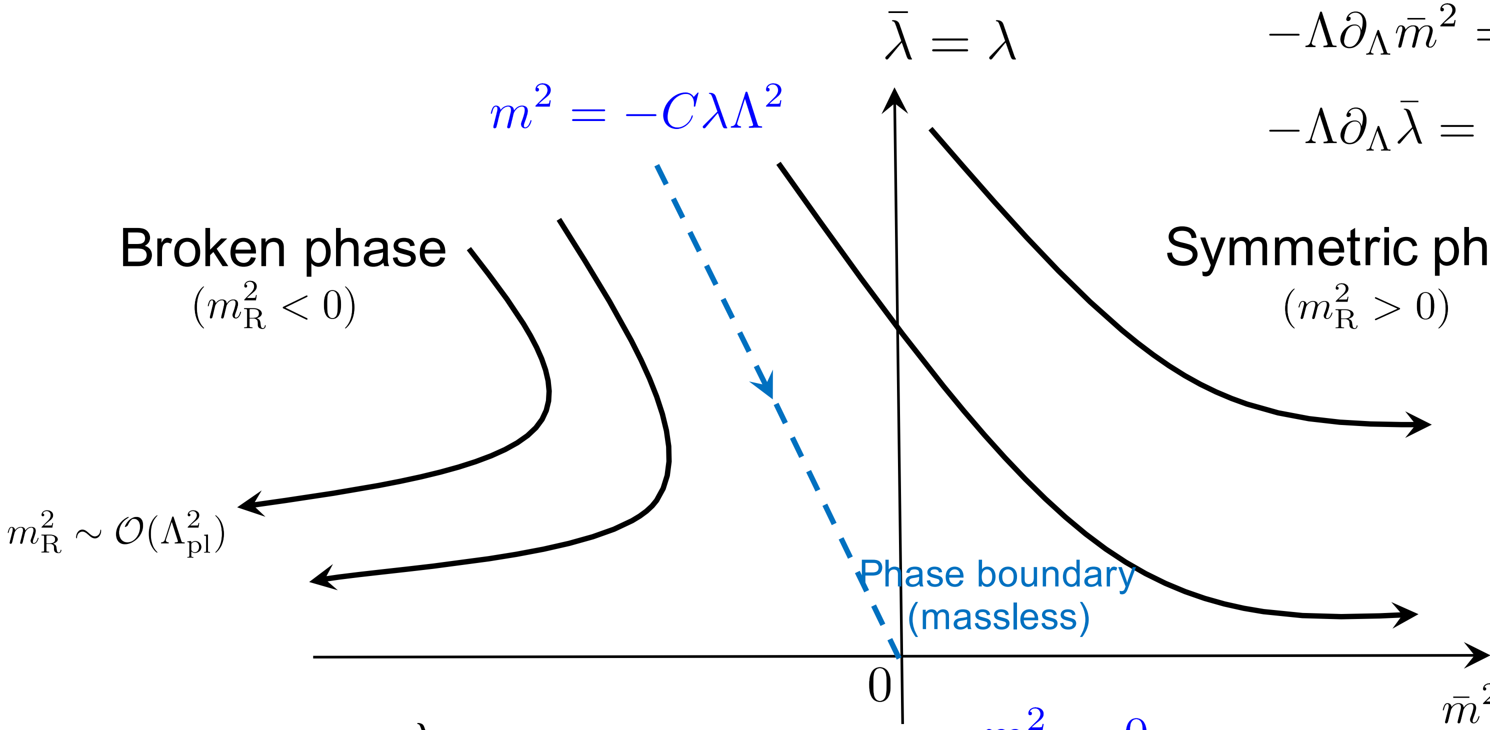
Fine-tuning problem in viewpoint of Wilson's RG

[Cf. H. Aoki, S. Iso, '14]

$$\mathcal{L}_{\mathrm{bare}}|_{\Lambda=\Lambda_{\mathrm{pl}}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4}\phi^4$$

$$-\Lambda \partial_\Lambda \bar{m}^2 = \beta_{\bar{m}^2}(\bar{m}^2, \bar{\lambda})$$

$$-\Lambda \partial_\Lambda \bar{\lambda} = \beta_{\bar{\lambda}}(\bar{m}^2, \bar{\lambda})$$



$$m_{\mathrm{R}}^2 = m_0^2 + \frac{\lambda}{16\pi^2}\Lambda_{\mathrm{pl}}^2$$

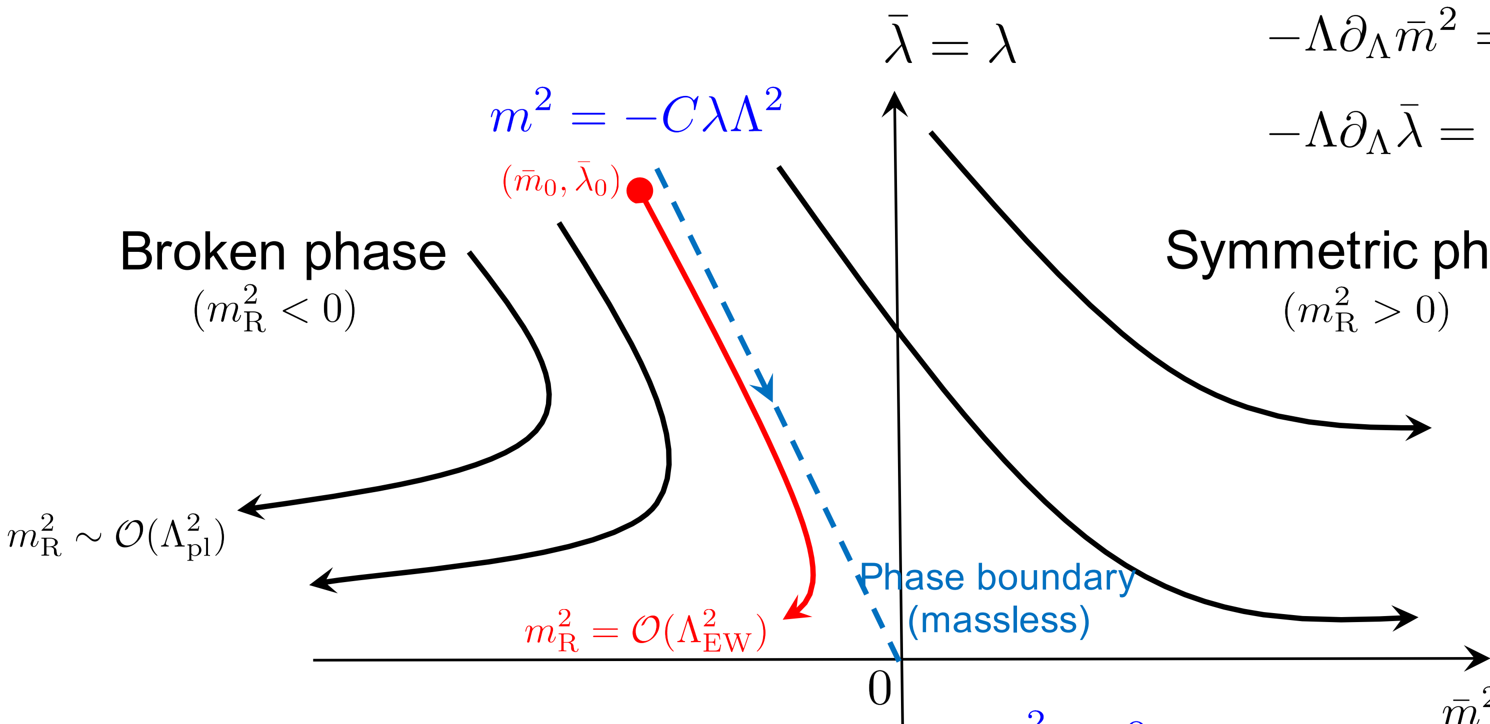
Fine-tuning problem in viewpoint of Wilson's RG

[Cf. H. Aoki, S. Iso, '14]

$$\mathcal{L}_{\rm bare}|_{\Lambda=\Lambda_{\rm pl}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4}\phi^4$$

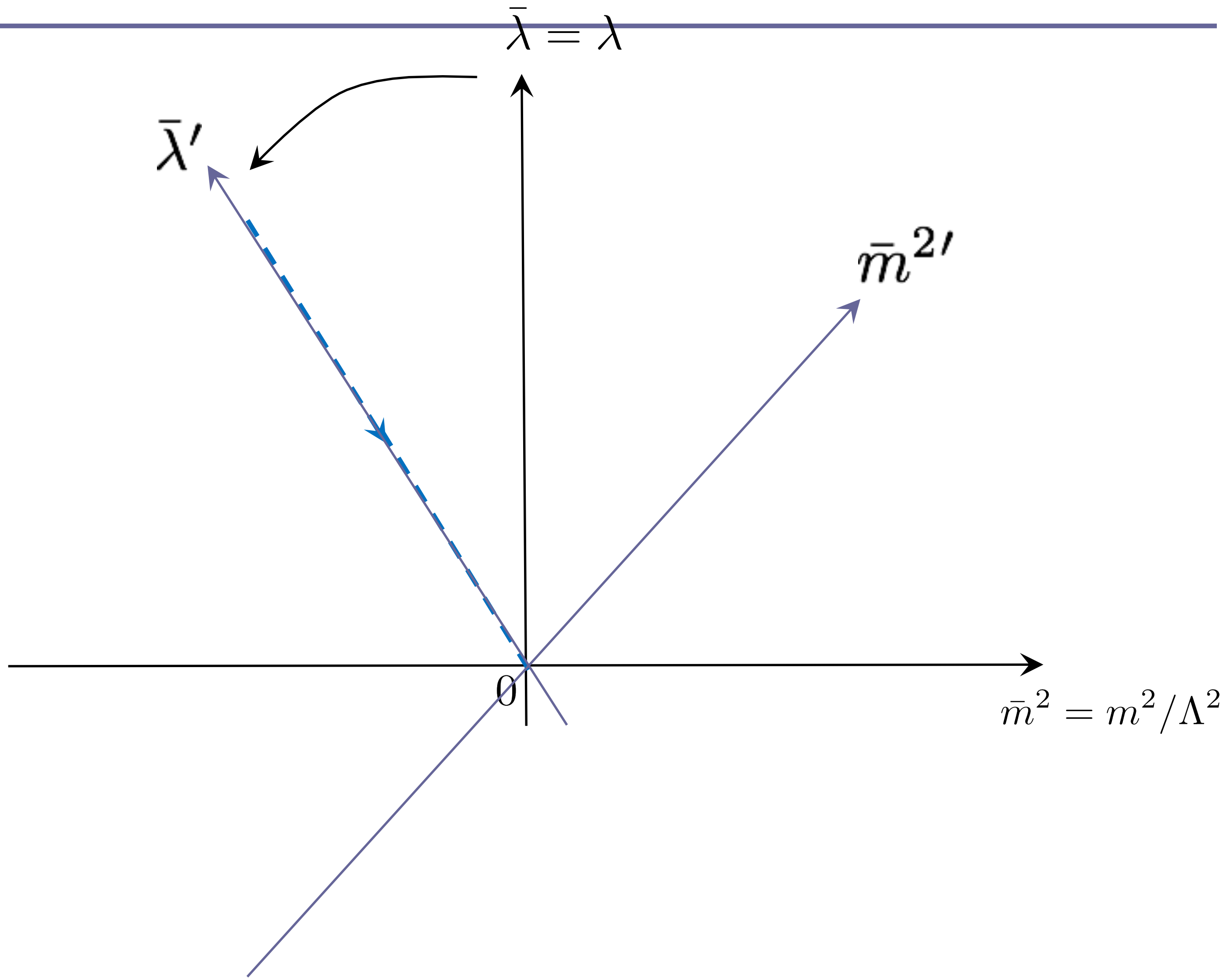
$$-\Lambda\partial_\Lambda \bar{m}^2 = \beta_{\bar{m}^2}(\bar{m}^2, \bar{\lambda})$$

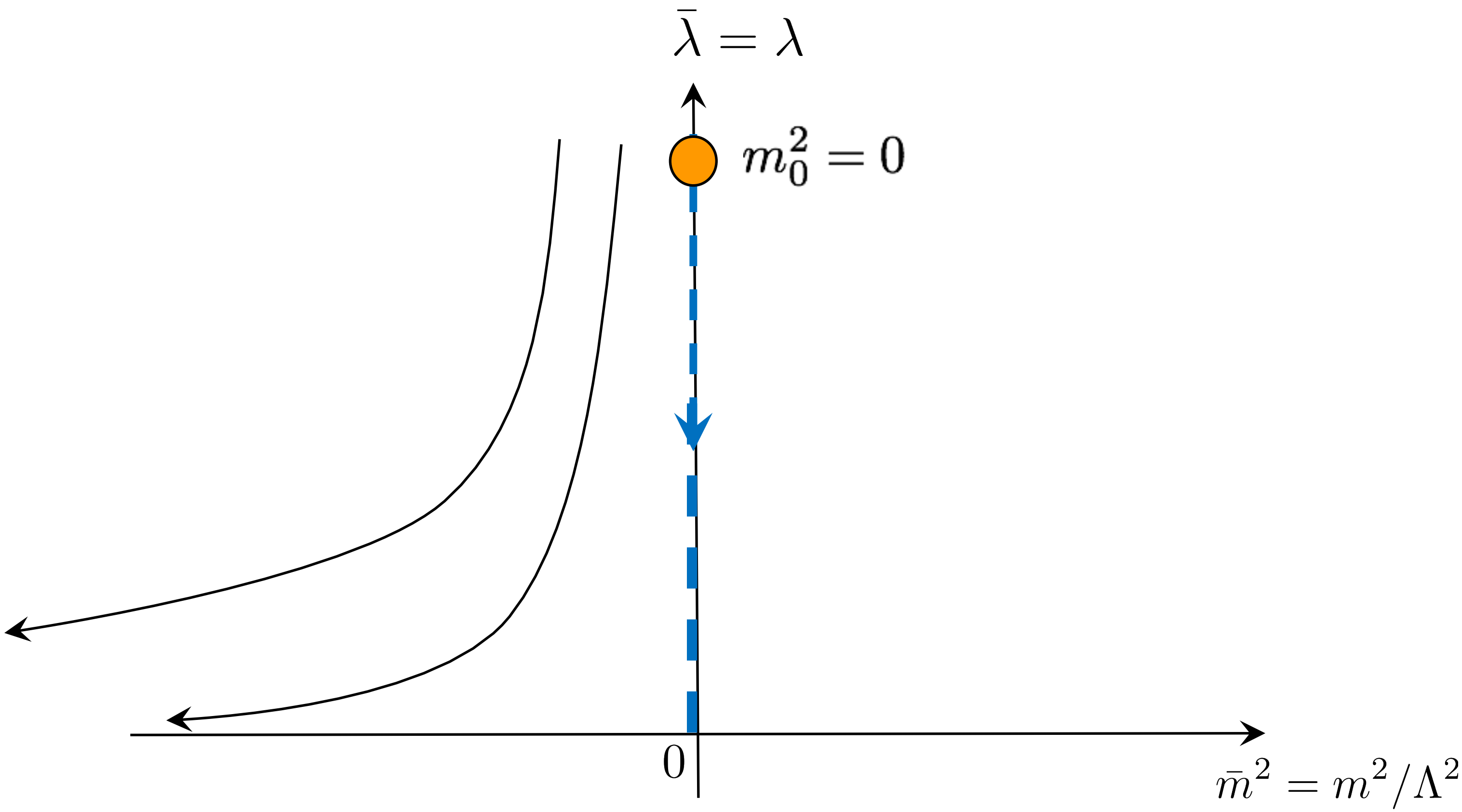
$$-\Lambda\partial_\Lambda \bar{\lambda} = \beta_{\bar{\lambda}}(\bar{m}^2, \bar{\lambda})$$



$$m_R^2 = m_0^2 + \frac{\lambda}{16\pi^2}\Lambda_{\rm pl}^2$$

Fine-tune problem = Why is the Higgs close to critical?


$\bar{\lambda} = \lambda$
$\bar{\lambda}'$
$\bar{m}^{2\prime}$
0
$\bar{m}^2 = m^2/\Lambda^2$



The classical scale invariance = The bare mass is exactly put on the critical line.

The massless theory (critical theory) is realized.

The classical scale invariance makes the Higgs critical.

Classical scale invariance

- The classical scale invariance prohibits m_0.
  - Boundary condition: $m_0 = m(\Lambda_{\rm pl}) = 0$
- The origin of observed mass is radiative corrections with TeV scale.

$$m_R^2 = \frac{\lambda'}{16\pi^2} M^2 \log\left(\frac{\mu^2}{M^2}\right)$$

- The classical scale invariance is one of candidates for the solution of fine-tuning problem.

How to generate radiative corrections?

Advantages of our model

- The number of parameters is less.
- The mediator is the strongly interacting particle.
  - Observing the hidden sector is easier than other models such as the hidden (quark) model.
    - $< \bar{\psi}\psi > \to < S > \to m_H \to < h >$
    - $< S^\dagger S > \to m_H \to < h >$
  - The DM candidate is CP even.
    - c.f. The DM in hidden (quark) QCD is CP odd.
- Strong 1st order of EW phase transition can be realized.(will see later)

Where is the vacuum?

- Minimum of V_{MFA}; Solving gap equations:

$$\frac{\partial}{\partial S_i^a} V_{\mathrm{MFA}} = 0, \quad \frac{\partial}{\partial f} V_{\mathrm{MFA}} = 0, \quad \frac{\partial}{\partial H} V_{\mathrm{MFA}} = 0$$

- Three solutions:

i. $< S_i^a > \neq 0, < M^2 > = 0, G = 0$

ii. $< S_i^a > = 0, < M^2 > = 0$ $\longrightarrow$ $\langle V_{\mathrm{eff}} \rangle = 0$

iii. $< S_i^a > = 0, < M^2 > \neq 0, G > 0$ $\rightarrow$ $\langle V_{\mathrm{eff}} \rangle < 0$

$$M^2 = 2(N_{\mathrm{f}}\lambda_S + \lambda_S')f - \lambda_{HS} H^\dagger H$$

$$G = 4N_{\mathrm{f}}\lambda_H\lambda_S - N_{\mathrm{f}}\lambda_{HS}^2 + 4\lambda_H\lambda_S'$$

The solution (iii) is suitable.

How to evaluate physical values?

Review: T. Hatsuda and T. Kunihiro, Phys. Rep. 247 221 (1994)

- Mean-field approximation (MFA)
  - Many body system is reduced to 1 body system.
  - Methods:

1. Introduce a "BCS" vacuum $|\Omega\rangle$ and a mean field:

$$f_{ij} \equiv \langle\Omega|S_i^\dagger S_j|\Omega\rangle$$

2. Apply the following replacements to $\mathcal{L}_{\mathrm{eff}}$

$$(S_i^\dagger S_j)(S_j^\dagger S_i) \to :(S_i^\dagger S_j)(S_j^\dagger S_i): + 2f_{ij}(S_j^\dagger S_i) - |f_{ij}|^2$$

Normal ordering

3. We obtain

$$\langle\Omega| : \mathcal{L}_{\mathrm{Int}} : |\Omega\rangle = 0$$

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{MFA}} + :\mathcal{L}_{\mathrm{Int}}:$$

Mean-field approximation

- Bogoliubov-Valatin vacuum $|\Omega\rangle$

$$\langle\Omega|(S_i^\dagger S_j)|\Omega\rangle = f_0\delta_{ij} + Z_\sigma^{1/2}\delta_{ij}\sigma + Z_\phi^{1/2}t_{ji}^\alpha\phi^\alpha$$

$$\langle S_i S_j\rangle = \left\langle \sum_{a=1}^{N_c} S_i^a S_j^a \right\rangle$$

- Wick contractions

$$\langle\Omega| : \mathcal{O} : |\Omega\rangle = 0$$

$$(S_i^\dagger S_j) =: (S_i^\dagger S_j) : + f_{ij}$$

$$(S_i^\dagger S_j)(S_j S_i) =: (S_i^\dagger S_j)(S_j^\dagger S_i) : + 2f_{ij}(S_j^\dagger S_i) - |f_{ij}|^2$$

Mean-field approximation

- Lagrangian $\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm MFA} + \mathcal{L}_I$

$$\langle \Omega | \mathcal{L}_I | \Omega \rangle = 0$$

$$\begin{aligned}\mathcal{L}_{\rm MFA} = & (\partial^\mu S^\dagger \partial_\mu S) - M^2 (S_i^\dagger S_j) \\ & + N_f (N_f \lambda_S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda'_S}{2} Z_\phi \phi^\alpha \phi^\alpha \\ & - 2(N_f \lambda_S + \lambda'_S) Z_\sigma^{1/2} \sigma (S_i^\dagger S_i) - 2\lambda'_S Z_\phi^{1/2} (S_i^\dagger t_{ij}^\alpha \phi^\alpha S_j) \\ & + \lambda_{HS} (S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2\end{aligned}$$

- Constituent scalar mass

$$M^2 = 2(N_f \lambda_S + \lambda'_S) f - \lambda_{HS} H^\dagger H$$

Effective potential

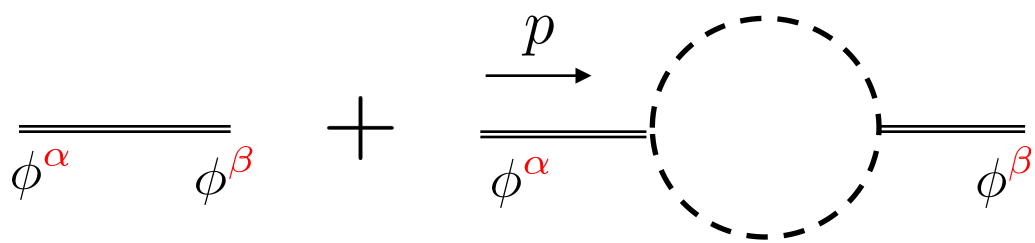
$$V_{\mathrm{MFA}} = M^2 (S_i^\dagger S_i) + \lambda_H (H^\dagger H)^2 - N_f (N_f \lambda_S + \lambda_S') f^2 + \frac{N_c N_f}{32\pi^2} M^4 \log \frac{M^2}{\Lambda_H^2}$$

$$H = \begin{pmatrix} \chi^+ \\ \langle h \rangle + h + i\chi^0 \end{pmatrix}$$

Mass of dark matter

- Mass = a pole of two point function
  - Inverse two point function of ϕ^α (dark matter)

$$\Gamma^{\alpha\beta}_{\phi\phi}(p^2) =$$

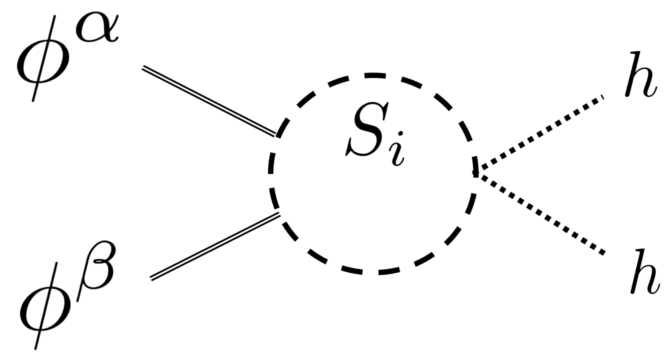


$$= \quad \delta^{\alpha\beta}\left[Z_\phi \lambda'_S \; + \; Z_\phi \lambda'^2_S N_c \Gamma(p^2)\right]$$

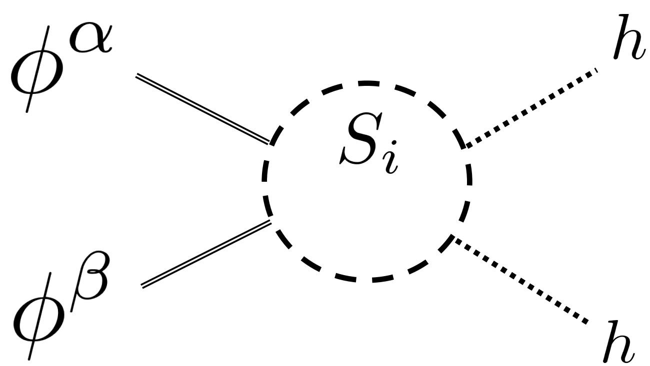
  - Find zero

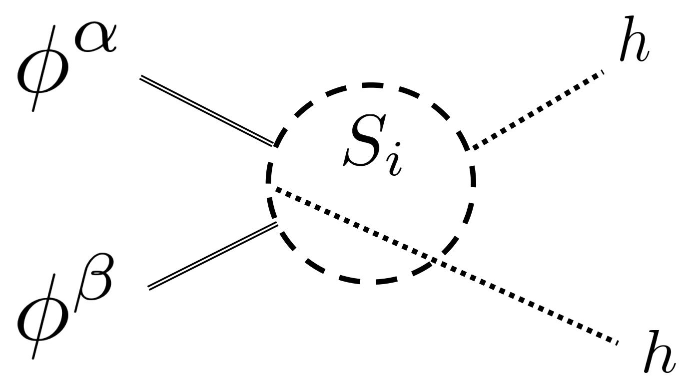
$$\Gamma^{\alpha\beta}_{\phi\phi}(p^2 = m^2_{\mathrm{DM}}) = 0$$

Coannahilation

$\kappa_{s(t)}\delta_{\alpha\beta} =$ 

$+\text{cross}$

$+$ 

$+$ 

$+\text{crosses}$

Velocity averaged annihilation cross section

$$\langle v\sigma\rangle = \frac{1}{32\pi m_{\rm DM}^3} \sum_{I=W,Z,t,h} (m_{\rm DM}^2 - m_I^2)^{1/2} a_I + \mathcal{O}(v^2)$$

$$a_{W(Z)} = 4(2)[\mathrm{Re}(\kappa_s)]^2 \Delta_h^2 m_{W(Z)}^4 \left(3 + 4\frac{m_{\rm DM}^4}{m_{W(Z)}^4} - 4\frac{m_{\rm DM}^2}{m_{W(Z)}^2}\right)$$

$$a_t = 24[\mathrm{Re}(\kappa_s)]^2 \Delta_h^2 m_t^2 (m_{\rm DM}^2 - m_t^2)$$

$$a_h = [\mathrm{Re}(\kappa_s)]^2 \left(1 + 24\lambda_H \Delta_h \frac{m_W^2}{g^2}\right)^2$$

$$\Delta_h = (4m_{\rm DM}^2 - m_h^2)^{-1}$$

Dark matter candidate is ϕ^{α}

- The excitation fields from the vacuum $< S^{\dagger}S >$
  - Assume the unbroken $\mathrm{U}(N_{\mathrm{f}})$ flavor symmetry:

$$\langle\Omega|(S_i^{\dagger}S_j)|\Omega\rangle = f_0\delta_{ij} + \delta_{ij}Z_{\sigma}^{\frac{1}{2}}\sigma + t_{ji}^{\alpha}Z_{\phi}^{\frac{1}{2}}\phi^{\alpha}$$

- Mean-field Lagrangian (before integrating S)

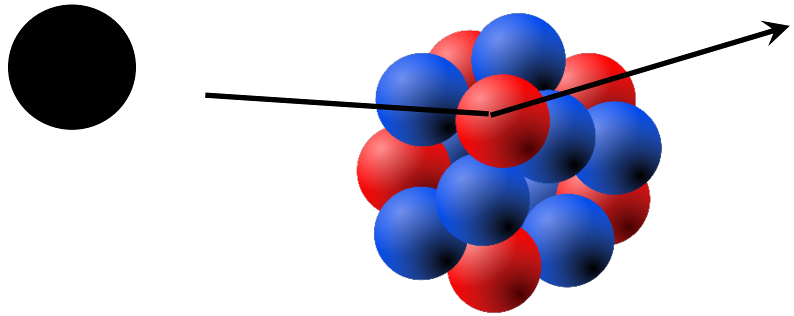
$$\mathcal{L}'_{\mathrm{MFA}} = (\partial_{\mu}S_i)^2 - M^2(S_i^{\dagger}S_i) + N_{\mathrm{f}}(N_{\mathrm{f}}\lambda_S + \lambda'_S)Z_{\sigma}\sigma^2 + \frac{\lambda'_S}{2}Z_{\phi}(\phi^{\alpha})^2$$

$$-2(N_{\mathrm{f}}\lambda_S + \lambda'_S)Z_{\sigma}^{1/2}\sigma(S_i^{\dagger}S_i) - 2\lambda'_S Z_{\phi}^{1/2}(S_i^{\dagger}t_{ij}^{\alpha}\phi^{\alpha}S_j)$$

$$+\lambda_{HS}(S_i^{\dagger}S_i)H^{\dagger}H - \lambda_H(H^{\dagger}H)^2$$

Direct detection

- Scattering off the Nuclei

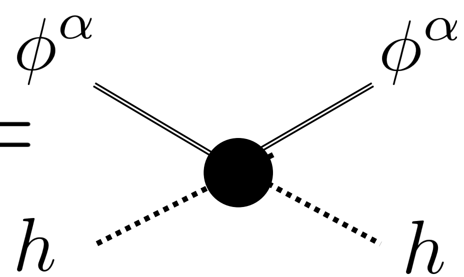


Inverse two-point function

$$\Gamma^{\alpha\beta}_{\phi\phi}(p^2 = m^2_{\mathrm{DM}}) = 0$$

  - Spin independent cross section

$$\sigma_{\mathrm{SI}} = \frac{1}{4\pi}\left(\frac{\kappa_t \hat{r} m_N^2}{m_{\mathrm{DM}} m_h^2}\right)^2 \left(\frac{m_{\mathrm{DM}}}{m_N + m_{\mathrm{DM}}}\right)^2$$

$\kappa_t =$ 

m_N: nucleon mass

$\hat{r}$: nucleonic matrix element $\quad \hat{r} \sim 0.3$

σ_{SI}

$$\sigma_{\mathrm{SI}} = \frac{1}{4\pi}\left(\frac{\kappa_t \hat{r} m_N^2}{m_{\mathrm{DM}} m_h^2}\right)^2 \left(\frac{m_{\mathrm{DM}}}{m_N + m_{\mathrm{DM}}}\right)^2$$

$$\hat{r} \sim 0.3$$

Dark matter relic abundance

- DM relic abundance

$$\Omega\hat{h}^2 = (N_{\rm f}^2 - 1)\frac{Y_\infty s_0 m_{\rm DM}}{\rho_c/\hat{h}^2}$$

  - Entropy density

$$s_0 = 2890\ \mathrm{cm}^{-3}$$

  - Critical density/Hubble parameter

$$\rho_c/\hat{h}^2 = 1.05\times 10^{-5}\ \mathrm{GeV\ cm}^{-3}$$

  - DM number density

$$g_* = 106.75 + N_{\rm f}^2 - 1$$

$$\frac{dY}{dx} = -0.264 g_*^{1/2}\frac{m_{\rm DM}M_{\rm pl}}{x^2}\langle\sigma v\rangle(Y^2 - \bar{Y}^2)$$

At finite temperature

- Momentum integral

$$\int \frac{d^4p}{(2\pi)^4} f(p_0, \vec{p}) \Rightarrow T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} f(\omega_n, \vec{p})$$

  - Matsubara frequency

$$\omega_n = \begin{cases} 2n\pi T & \text{(boson loop)} \\ (2n+1)\pi T & \text{(fermion loop)} \end{cases}$$

Effective potential

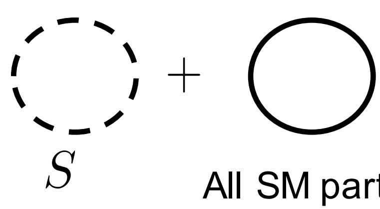
- There are four components.

$$V_{\text{eff}}(f, h; T) =$$

$$V_{\text{MFA}}(f, h) + V_{\text{CW}}(h)$$ Zero temp. part

$$+\underline{V_{\text{FT}}(f, h; T)} + V_{\text{RING}}(h; T)$$ Finite temp. part

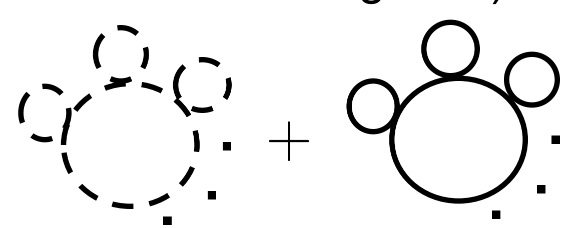
$$T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \Big[$$



$$\Big]$$

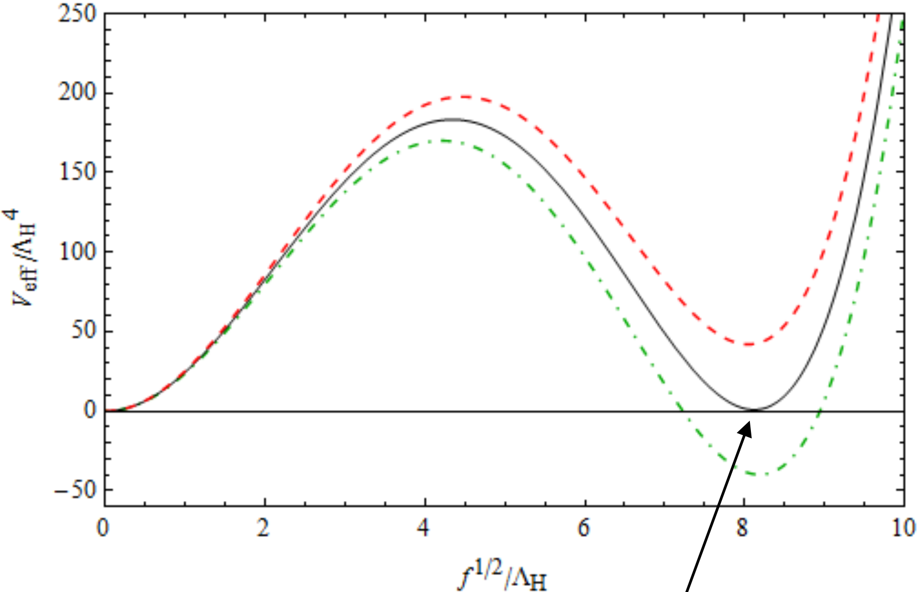
Summation of thermal mass

(remove the IR divergence)



Scale transition is strong 1st order.

J, Kubo and **M. Y.**, PTEP **2015** 093B01 (arXiv:1506.06460)



$T_{\rm c}^{\rm SS}/\Lambda_{\rm H} = 7.0$

$$\frac{\langle S^\dagger S\rangle^{1/2}}{\Lambda_H}$$