# Can massive primordial black holes be produced in mild waterfall hybrid inflation?

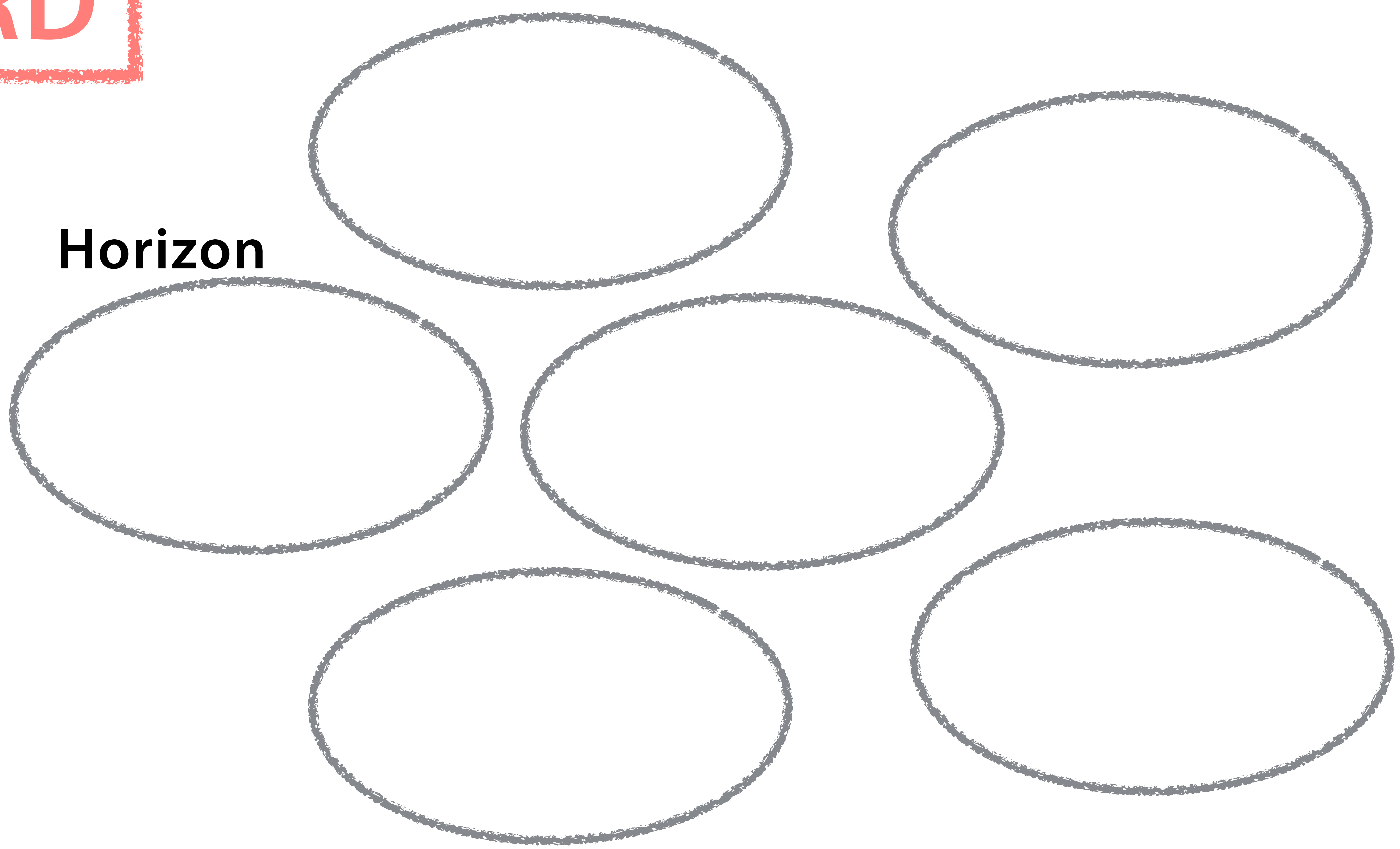
Yuichiro Tada (Kavli IPMU, ICRR)

w/ M. Kawasaki arXiv: 1512.03515

# Primordial Black Holes

Hawking 1971

RD



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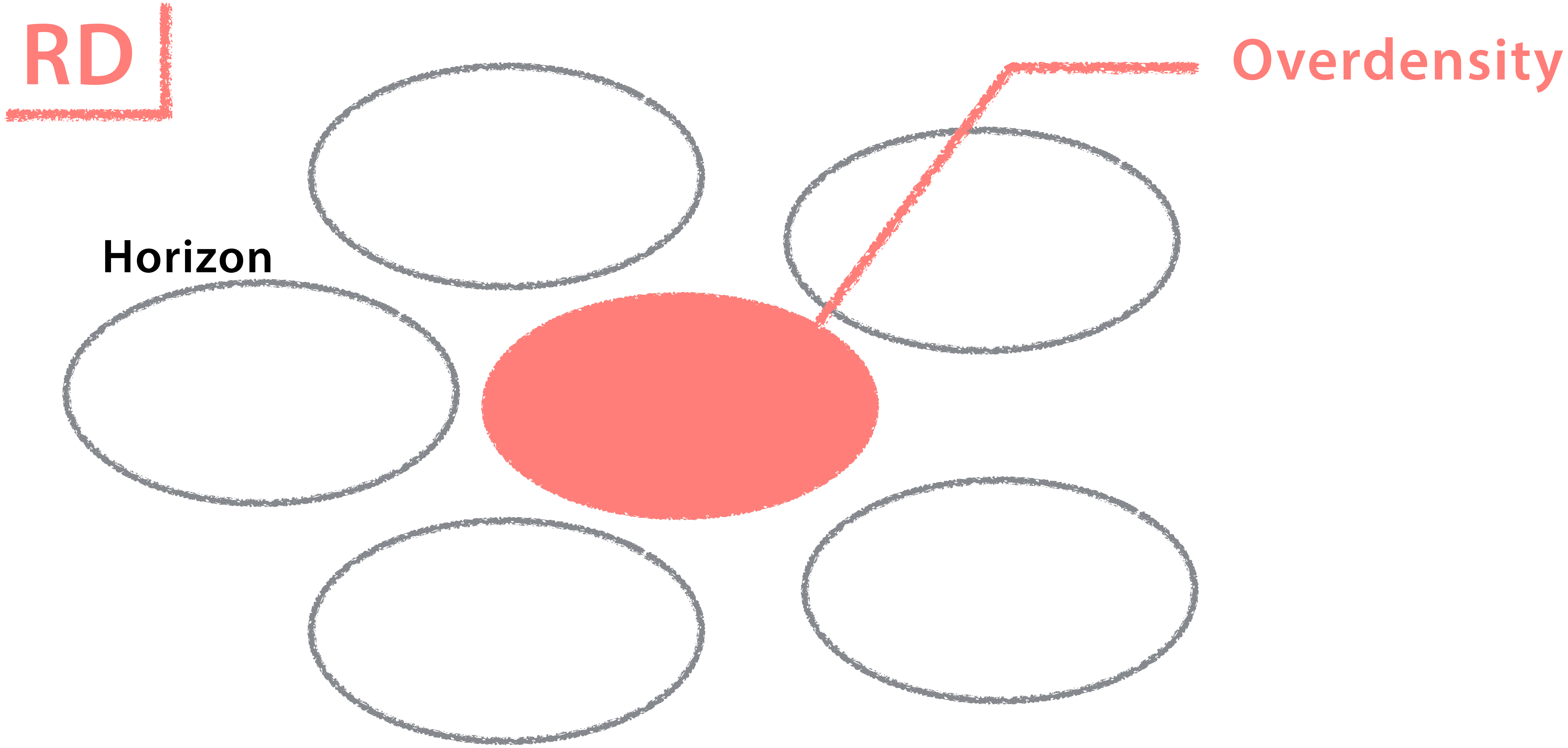

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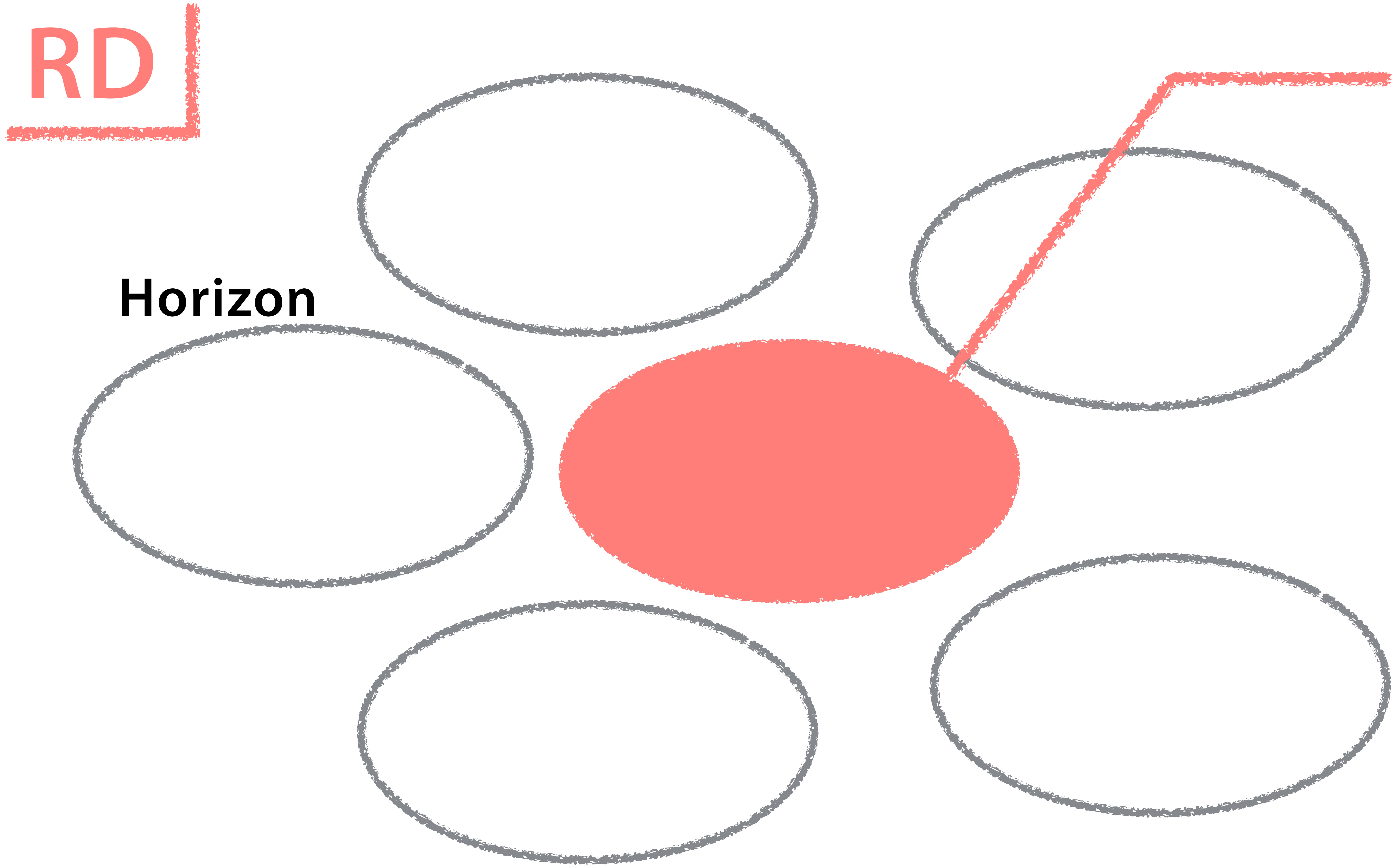


curvature perturbation

$$\zeta \gtrsim \zeta_c \sim 1$$

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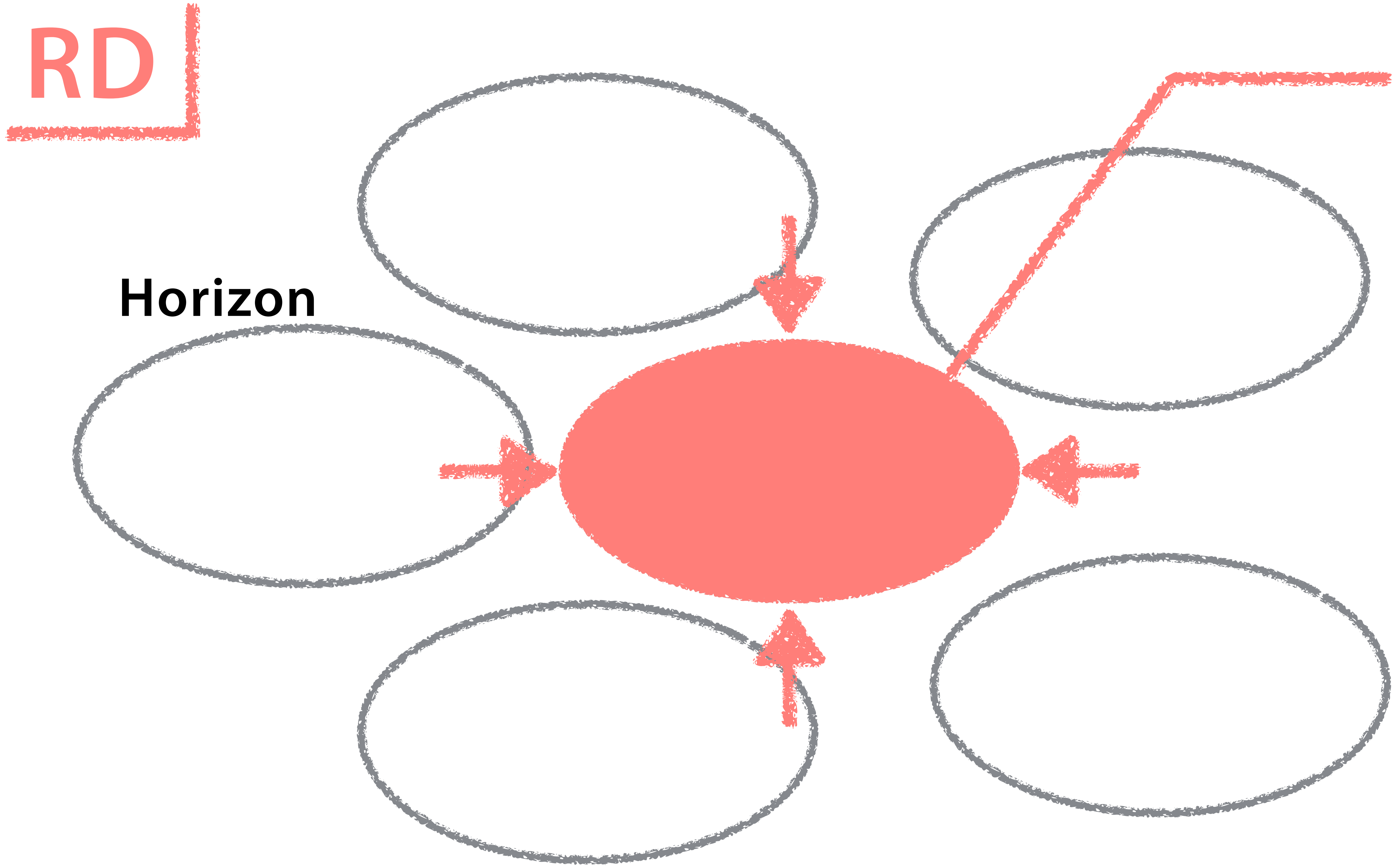


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$\zeta \gtrsim \zeta_c \sim 1$

↓

**Primordial Black Hole**

$$M_{\rm PBH} \sim \frac{M_p^2}{H_{\rm inf}} e^{2N}$$
$$= 10^{-29} M_\odot \left(\frac{10^9\,{\rm GeV}}{H_{\rm inf}}\right) e^{2N}$$

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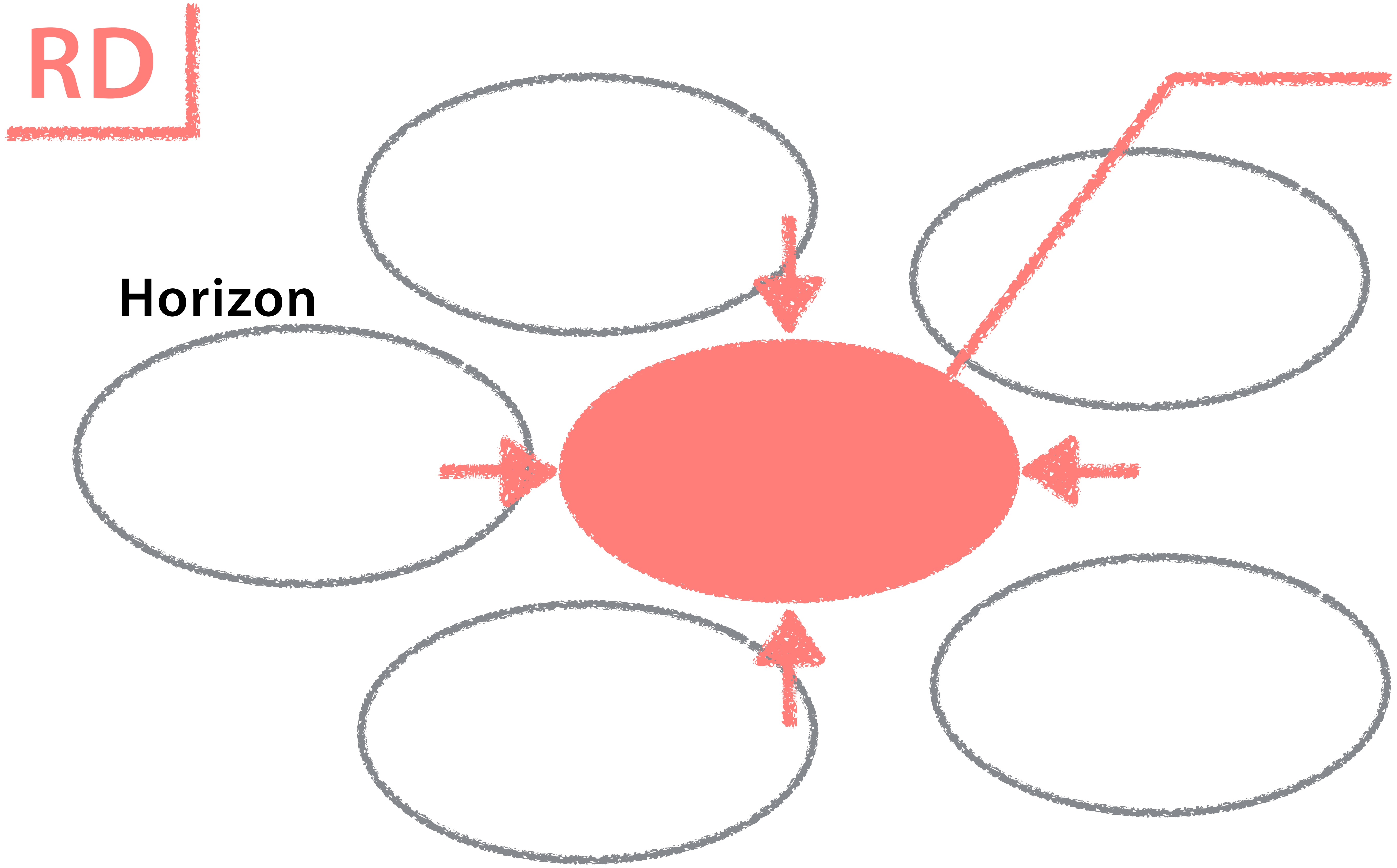


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**PBH rarity ↔ amplitude of fluctuations on small scale**

# Observational motivation

* Dark Matter (need NOT new particle!)

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## Did LIGO detect dark matter?

Simeon Bird,* Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski, Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess[1]

[1] *Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218, USA*

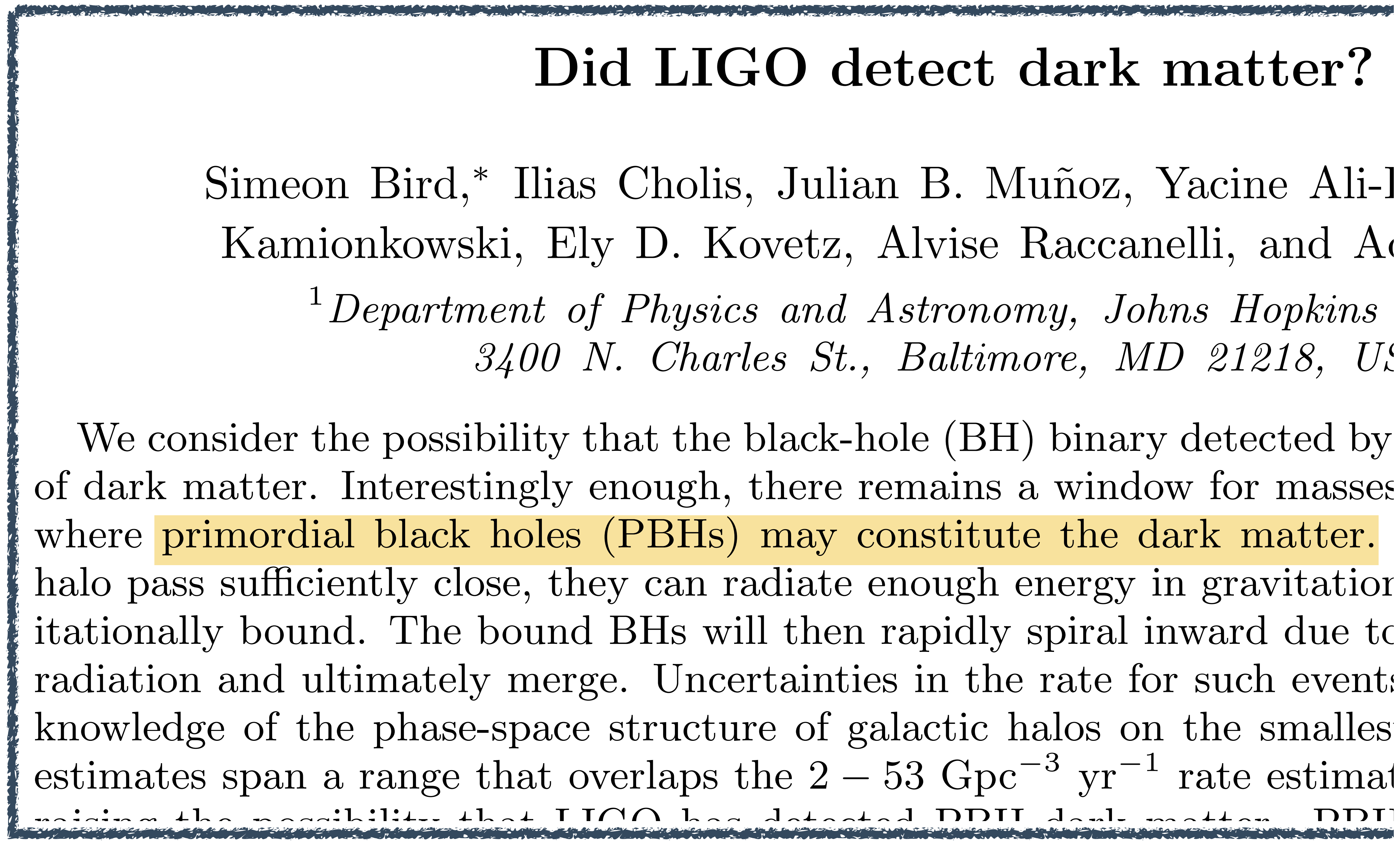
We consider the possibility that the black-hole (BH) binary detected by LIGO may be a signature of dark matter. Interestingly enough, there remains a window for masses $10\,M_\odot \lesssim M_{\rm bh} \lesssim 100\,M_\odot$ where primordial black holes (PBHs) may constitute the dark matter. If two BHs in a galactic halo pass sufficiently close, they can radiate enough energy in gravitational waves to become gravitationally bound. The bound BHs will then rapidly spiral inward due to emission of gravitational radiation and ultimately merge. Uncertainties in the rate for such events arise from our imprecise knowledge of the phase-space structure of galactic halos on the smallest scales. Still, reasonable estimates span a range that overlaps the $2-53$ Gpc$^{-3}$ yr$^{-1}$ rate estimated from GW150914, thus raising the possibility that LIGO has detected PBH dark matter. PBH mergers are likely to be

# Observational motivation

## ✱ Dark Matter (need NOT new particle!)

**GW by LIGO !!** (PRL.116.061102)

from merger of BH (~30 $M_\odot$) binary



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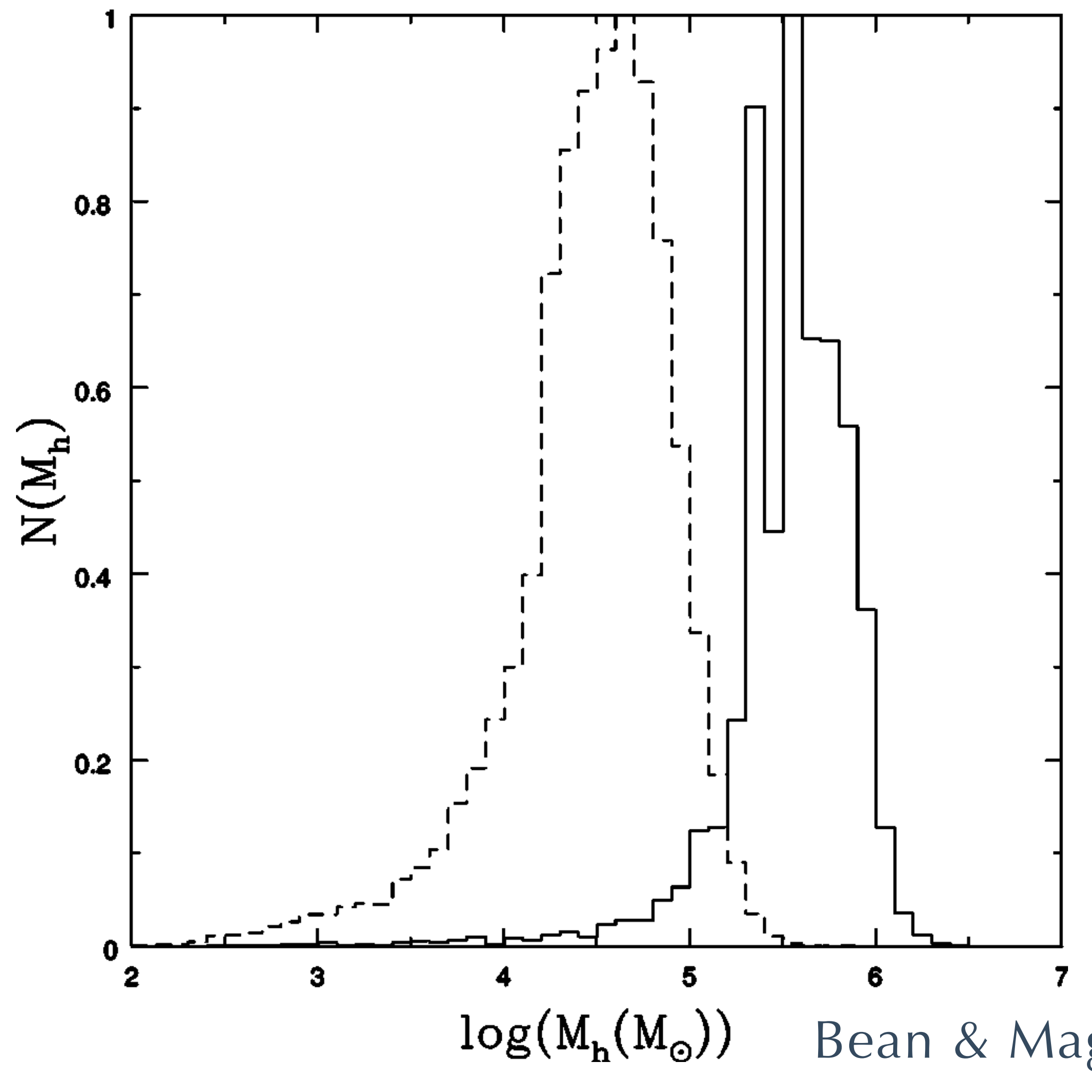
## ✱ SuperMassive Black Hole

- Almost all galaxies are thought to possess one or a few **SMBHs (~$10^{6\text{–}9.5}$ $M_\odot$)** in their centers
- They've been found even **at high redshift ($z$~6–7)** whose formations are difficult to be explained astrophysically. (Pop-III? Direct Collapse BH?)

Haiman & Loeb 2001
Made & Rees 2001

Bromm & Loeb 2003

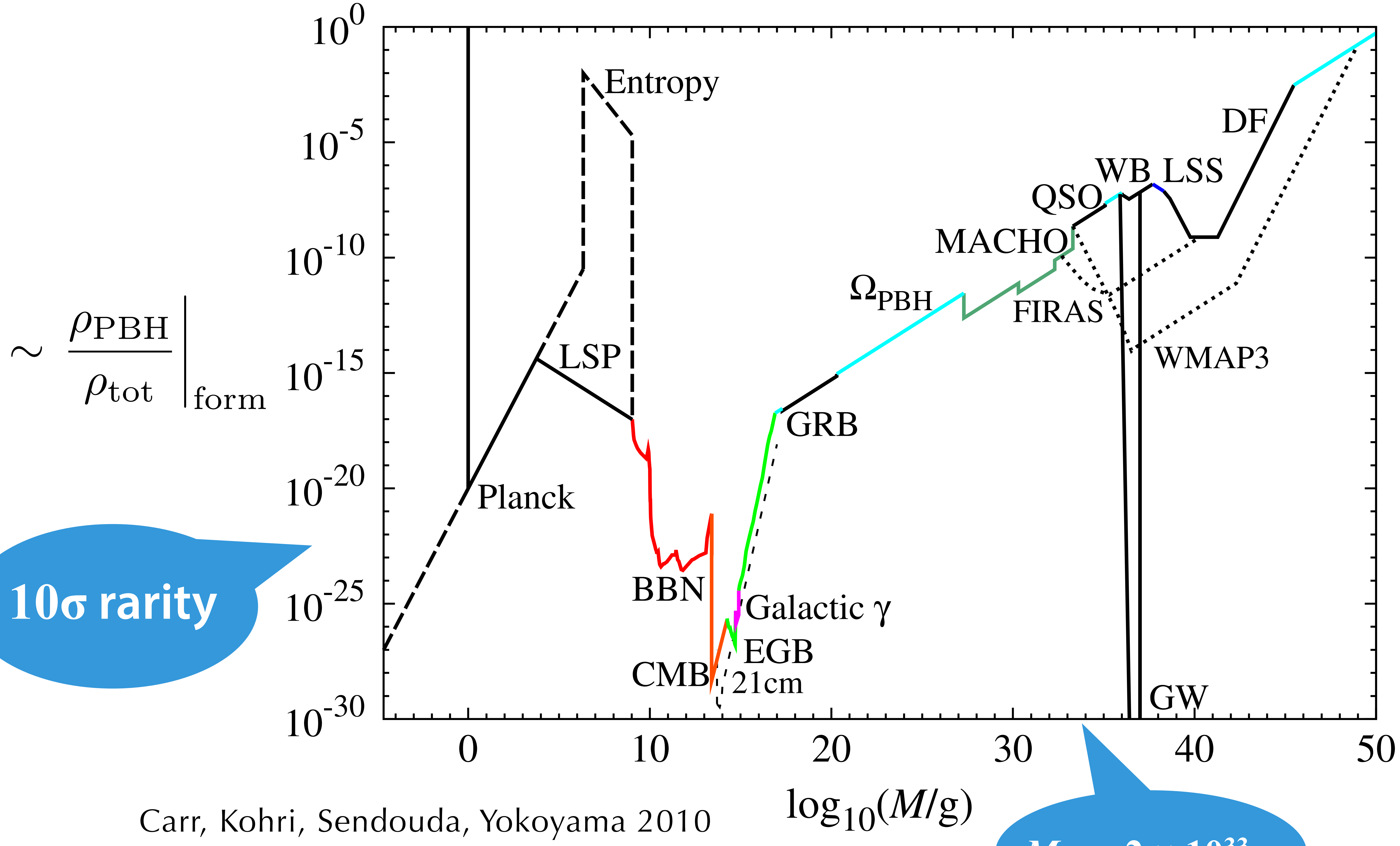



Bean & Magueijo 2002

**$M_{PBH}$ ~ $10^5 M_\odot$ can explain current SMBHs**

# Theoretical motivation

**PBH has NOT been detected**




**10σ rarity**

$M_\odot = 2 \times 10^{33}$ g

Carr, Kohri, Sendouda, Yokoyama 2010

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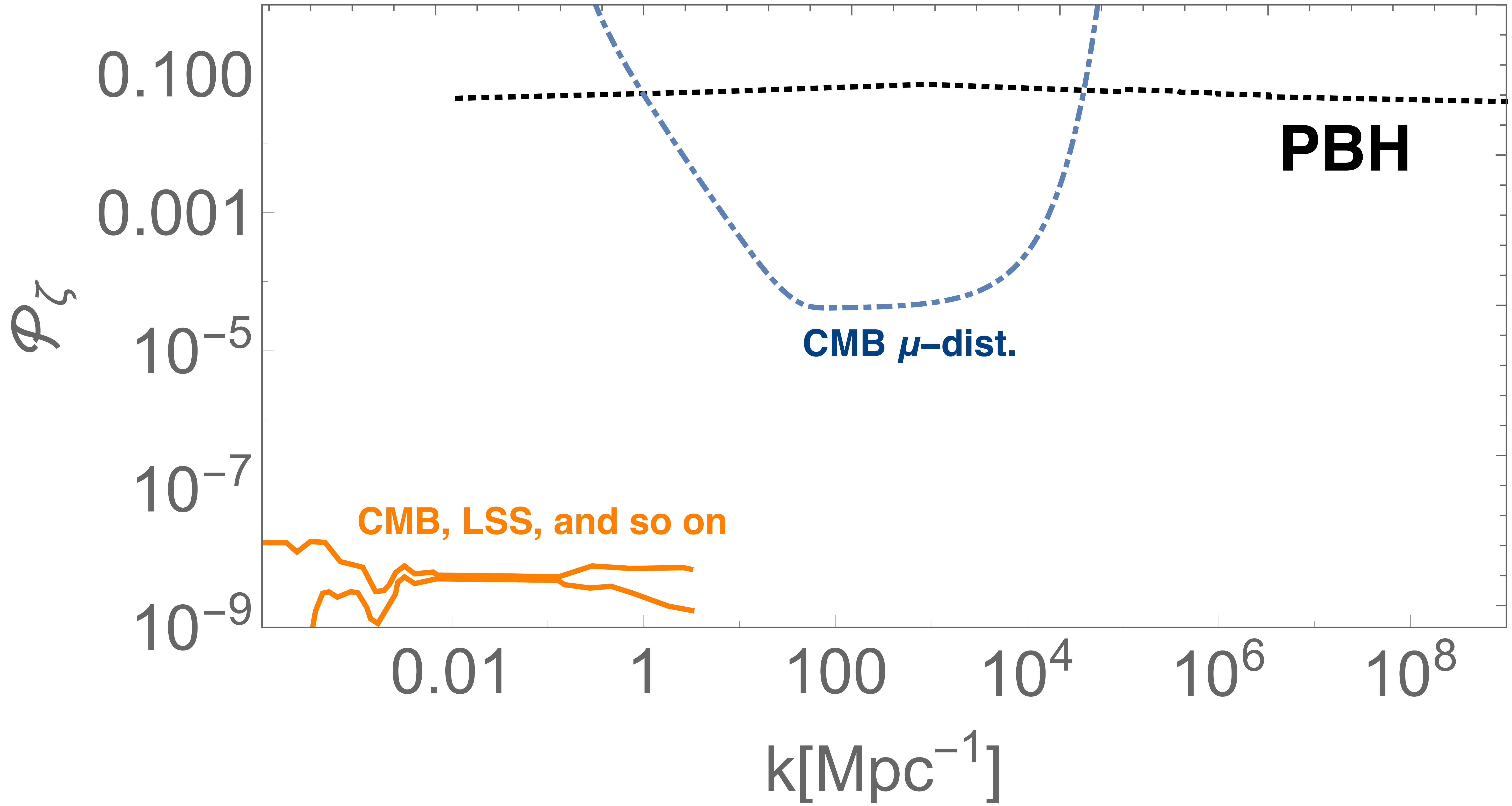
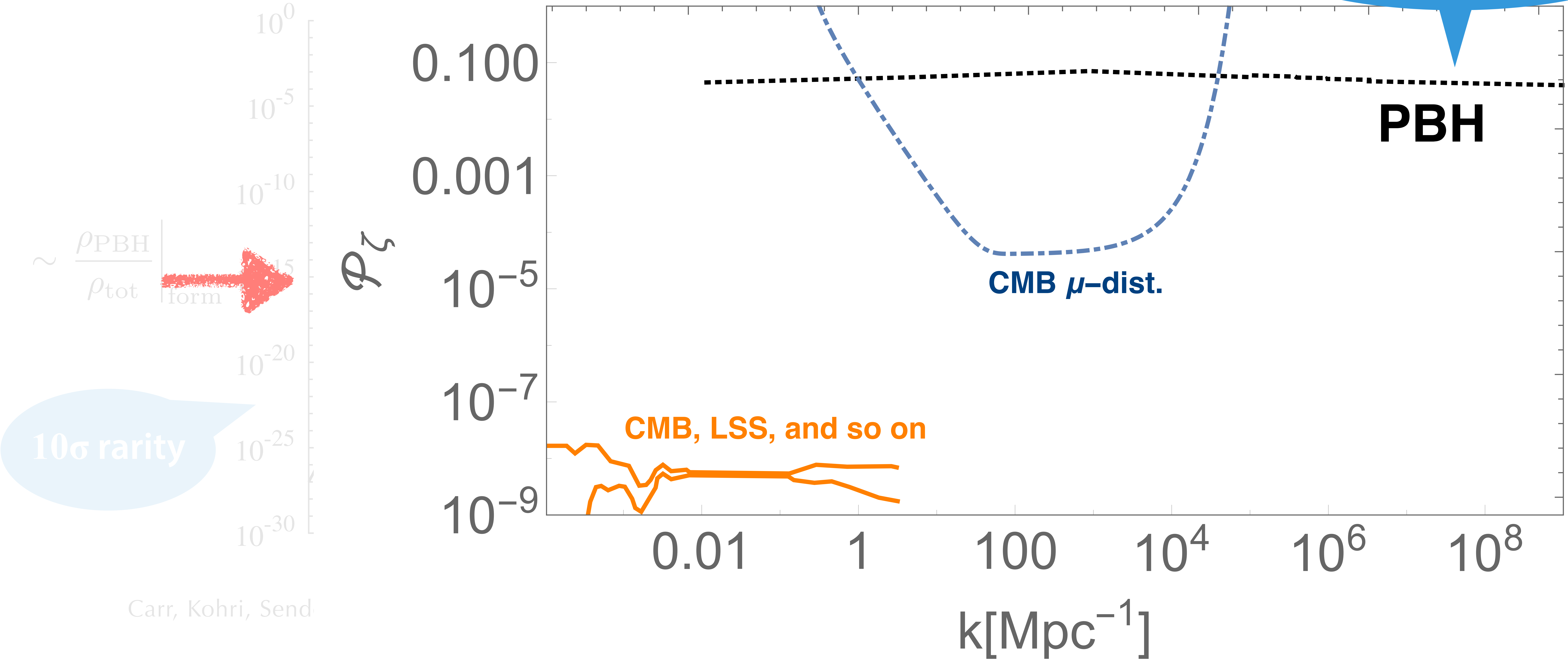


# Theoretical motivation

PBH has NOT been detected

$\sigma_\zeta^2 \lesssim (\zeta_c/10)^2 \sim 0.01$



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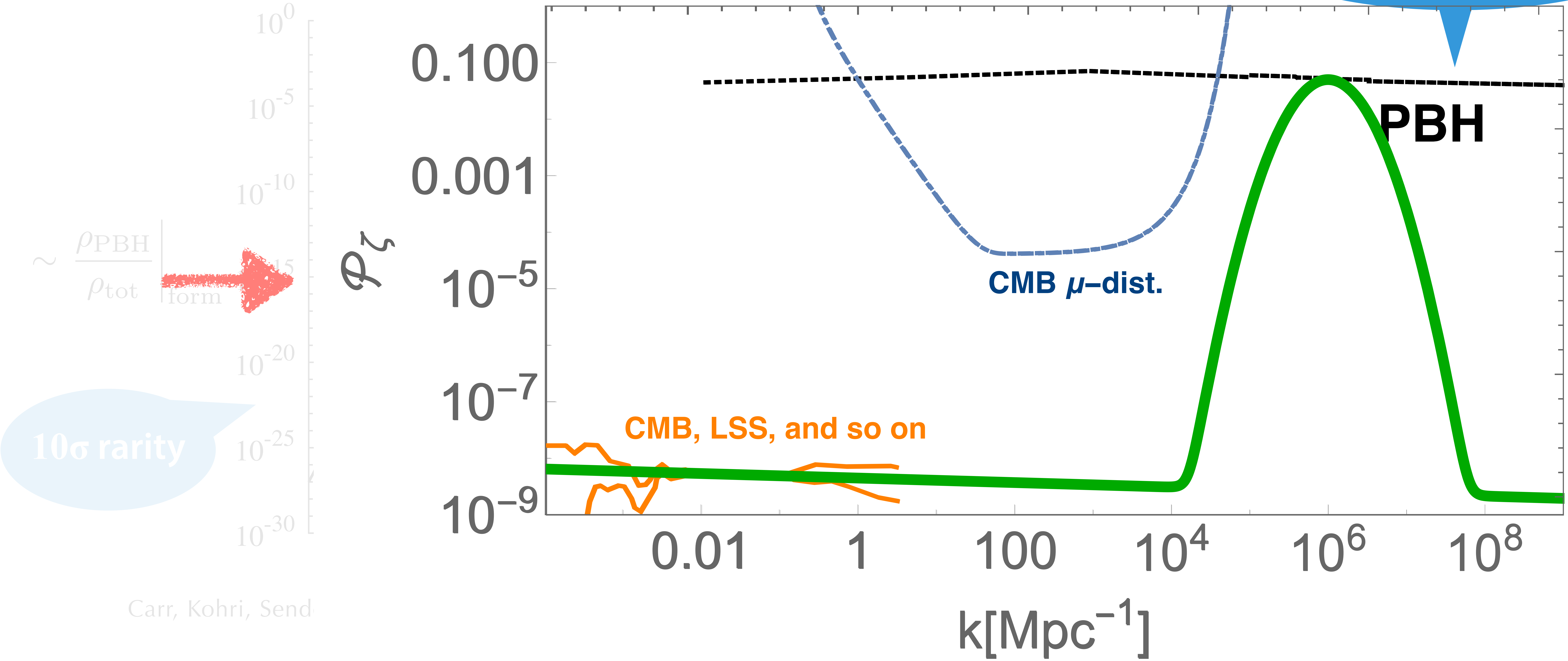
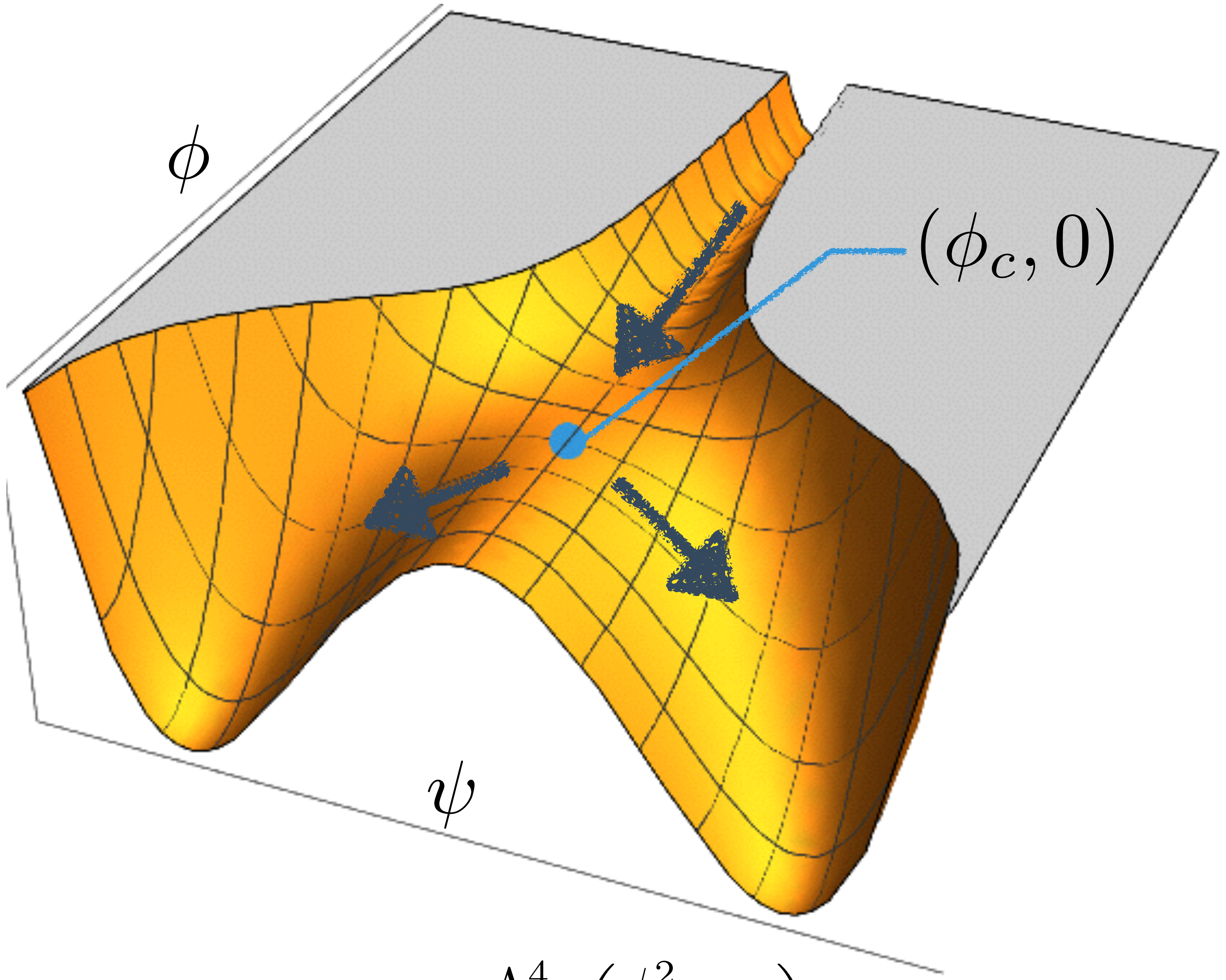


# Hybrid Inflation Linde 1994

$$V(\phi,\psi) = V(\phi) + \Lambda^4\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\Lambda^4\frac{\phi^2\psi^2}{\phi_c^2 M^2}$$




$$m_\psi^2|_{\psi\sim 0} = V_{\psi\psi}|_{\psi\sim 0} = 2\frac{\Lambda^4}{M^2}\left(\frac{\phi^2}{\phi_c^2}-1\right)$$

*** instant waterfall**

Chaotic IC + Small Field

$$V(\phi) = \begin{cases} \frac{1}{2}m^2\phi^2 & \text{original, blue-tilted} \\ \log|\phi/\phi_c| & \text{SUSY-flat + CW, } n_s \sim 0.98 \text{ Dvali, Shafi, Schaefer 1994} \\ \log|\phi/\phi_c| - \mathrm{Re}(\phi) & \text{SUSY breaking, } n_s \text{ can be 0.96 Buchmuller et. al. 2000-} \end{cases}$$

*** long waterfall ($N_{\rm water} > 60$)** Clesse 2011

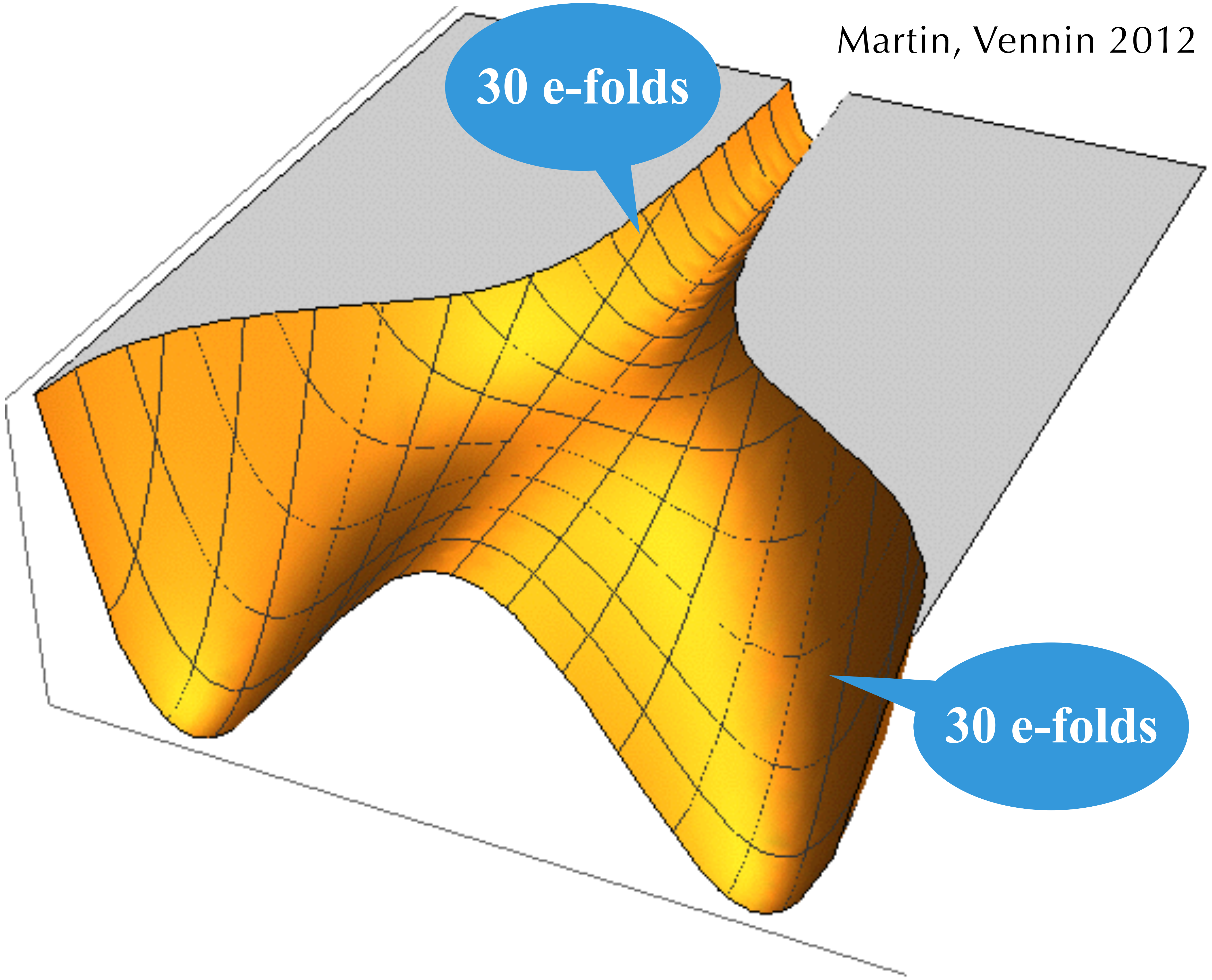
Relax the IC problem of Hilltop Inflation

# Mild Waterfall Case (half & half)

## Half & Half case and Massive PBH

Garcia-Bellido, Linde, Wands 1996
Lyth 2010, 2012
Bugaev, Klimai 2011, 2012
Clesse, Garcia-Bellido 2015
⋮

Martin, Vennin 2012




Perturbations become large around $\phi_c$ because of the flatness of the potential.

+

Following inflation enlarges the perturbation scale to make PBH massive.

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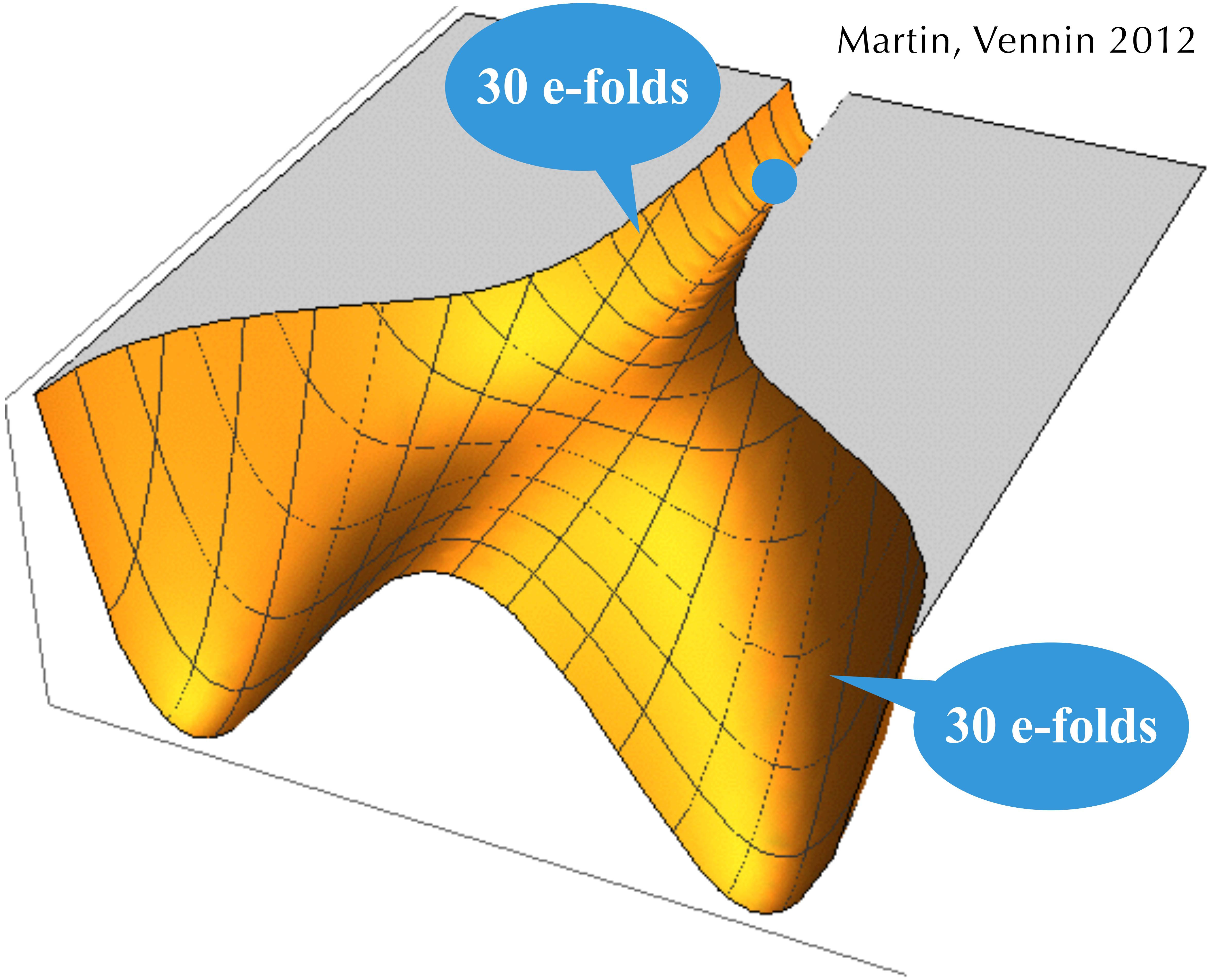


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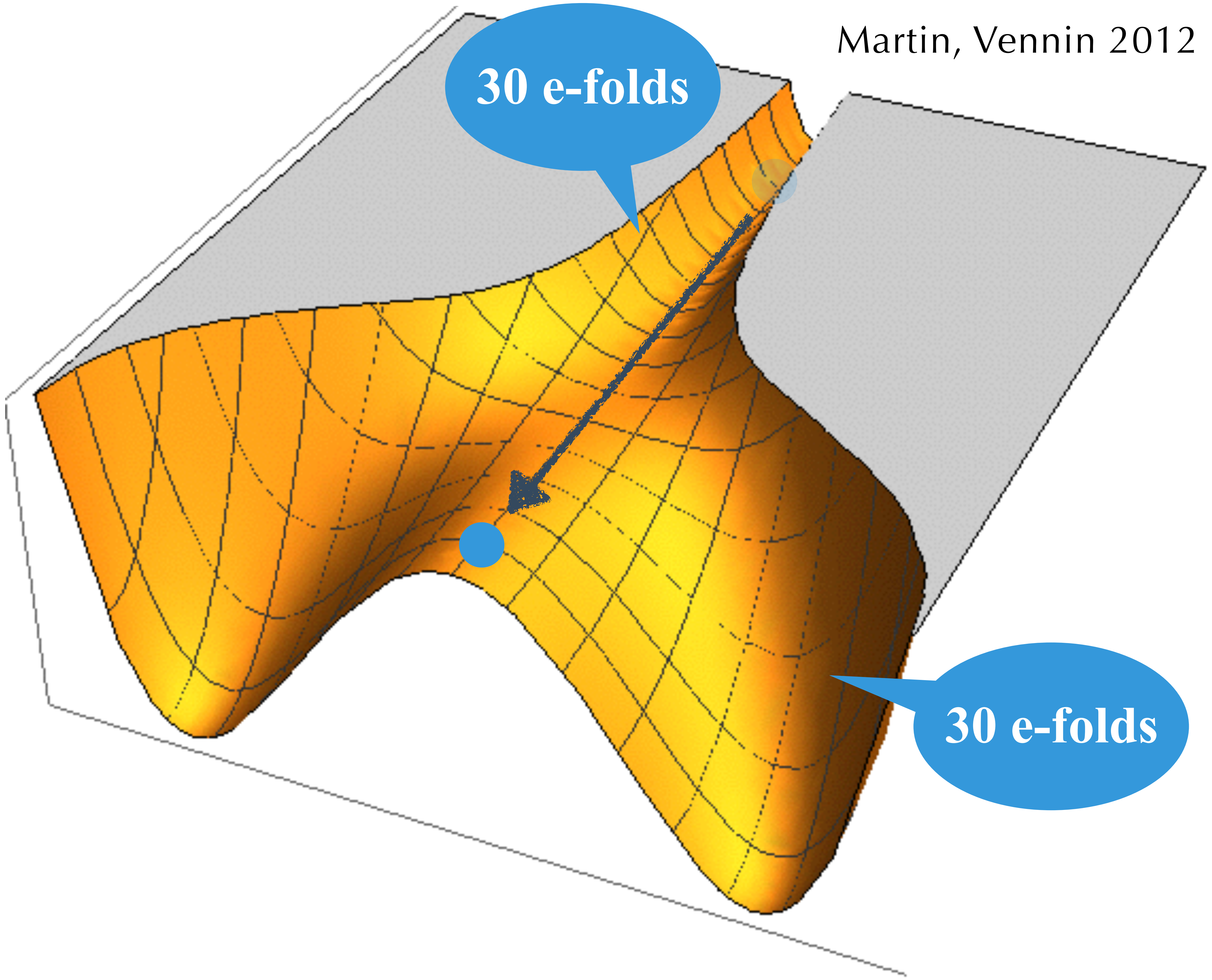


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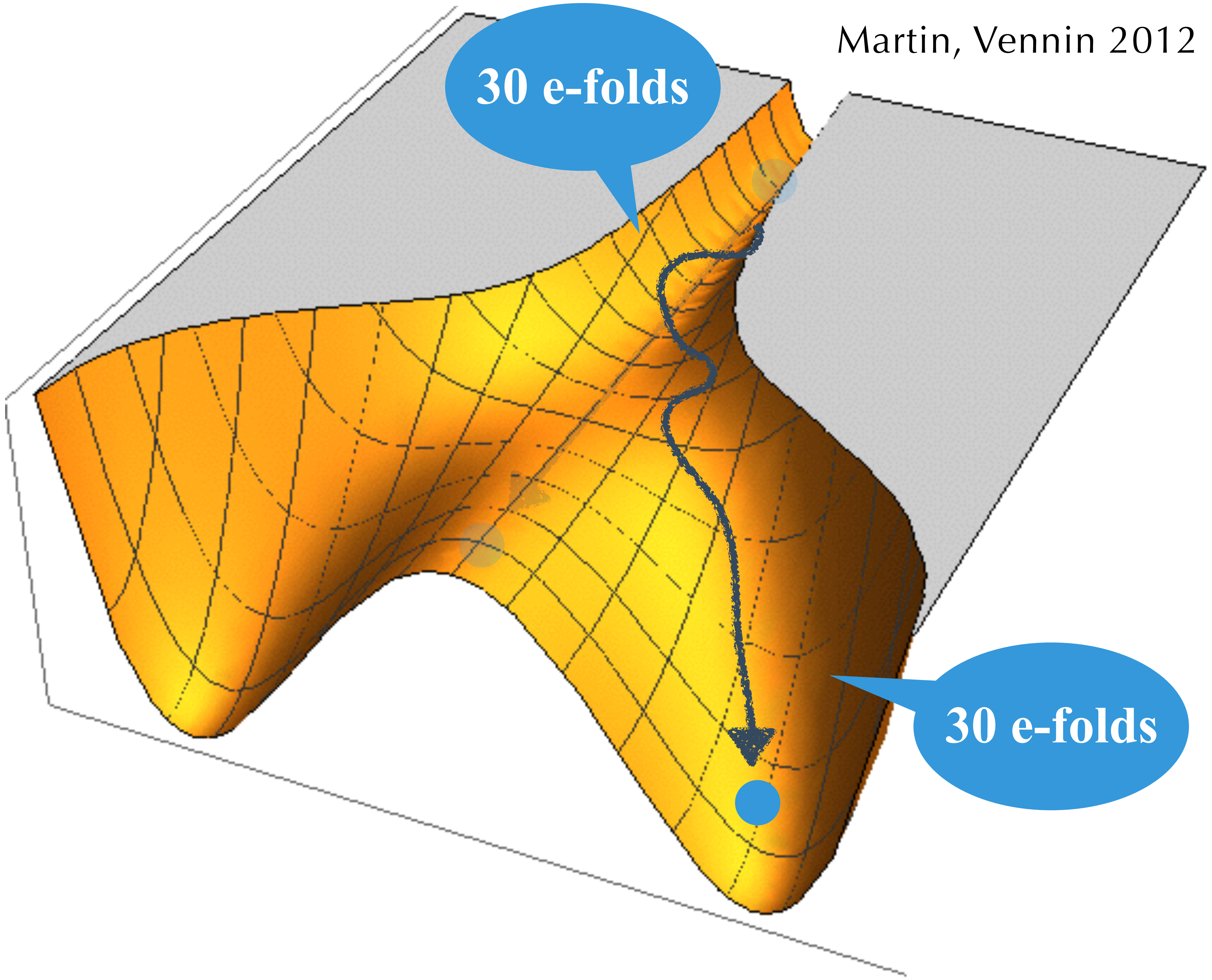

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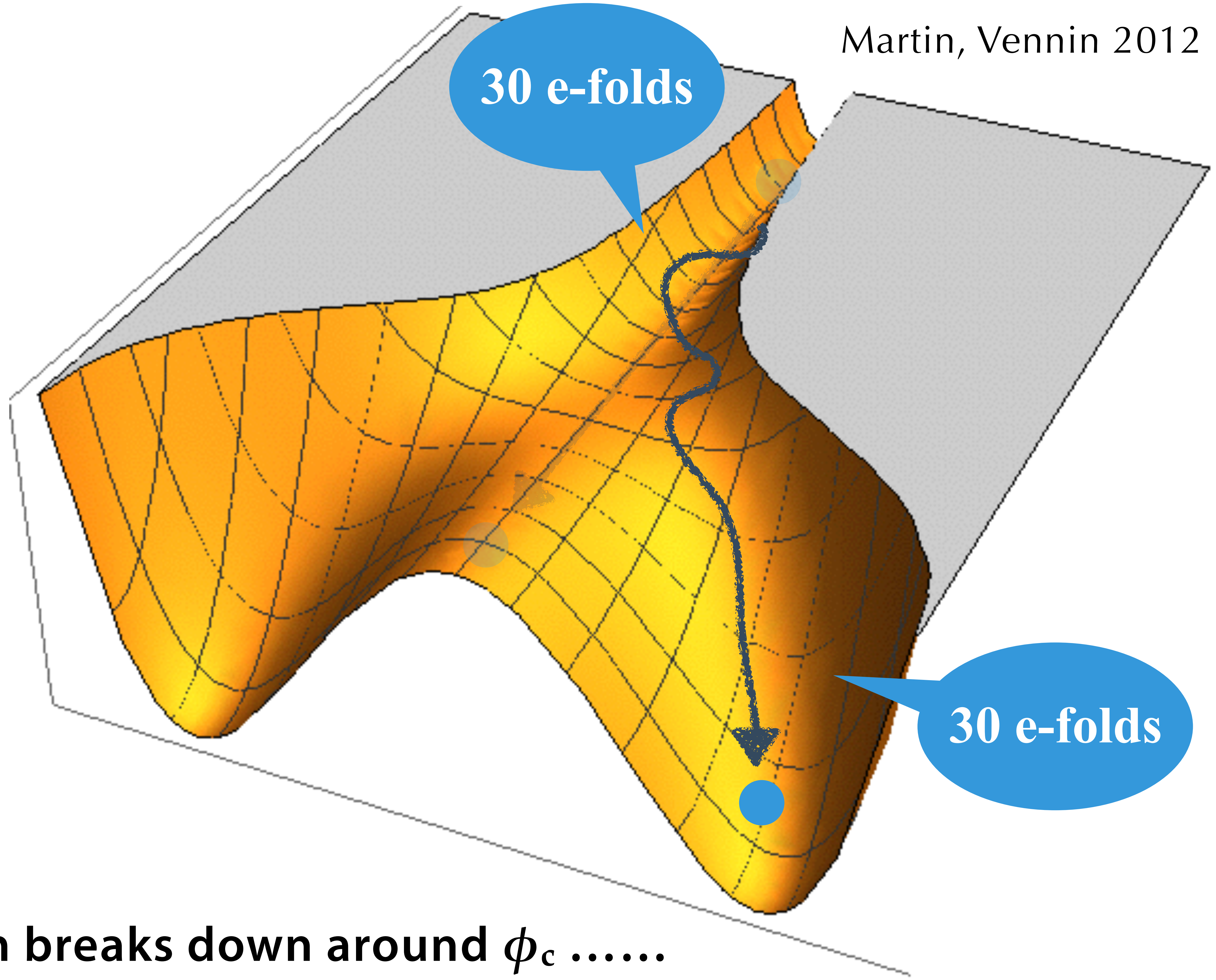


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Following inflation enlarges the perturbation scale to make PBH massive.

**However the perturbative expansion breaks down around $\phi_c$ ……**

→ Numerical calculation in non-perturbative way with Stochastic formalism!

*Short break*

# Stochastic formalism

Starobinsky 1986

**classical b.g. field**

coarse-grained on superhorizon scale

$$\phi_{\rm IR}(t,\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\theta(\epsilon aH - k)\phi_{\mathbf{k}}(t)\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\epsilon \ll 1$$

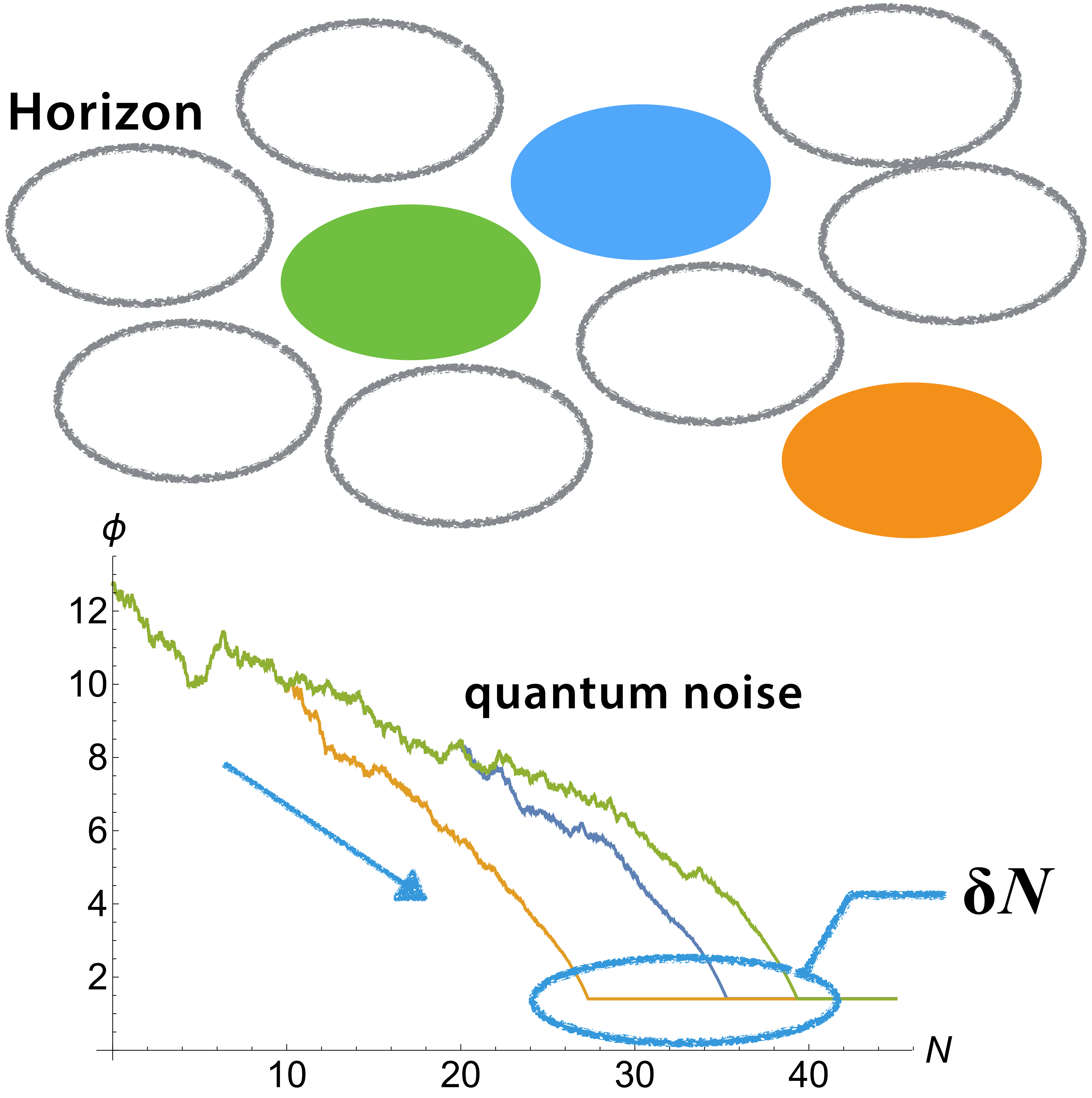
**EoM**

subhorizon → IR

$$3H\dot{\phi}_{\rm IR} + V' = 3H\xi(t,\mathbf{x}) = 3H\int\frac{\mathrm{d}^3k}{(2\pi)^3}\dot{\theta}(\epsilon aH - k)\phi_{\mathbf{k}}(t)\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\langle\xi(t,\mathbf{x})\xi(t',\mathbf{x}')\rangle \simeq H\mathcal{P}_\phi\delta(t-t')\theta(1-\epsilon aH|\mathbf{x}-\mathbf{x}'|)$$

$\xi$ can be interpreted as white and Hubble-patch-independent Gaussian noise



# Stochastic formalism
Starobinsky 1986

* $\delta N$-formalism Starobinsky 1985

$$ds^2 = \cdots + a_i^2 e^{2(N_0(t)+\delta N(t,\mathbf{x}))} d\mathbf{x}^2$$

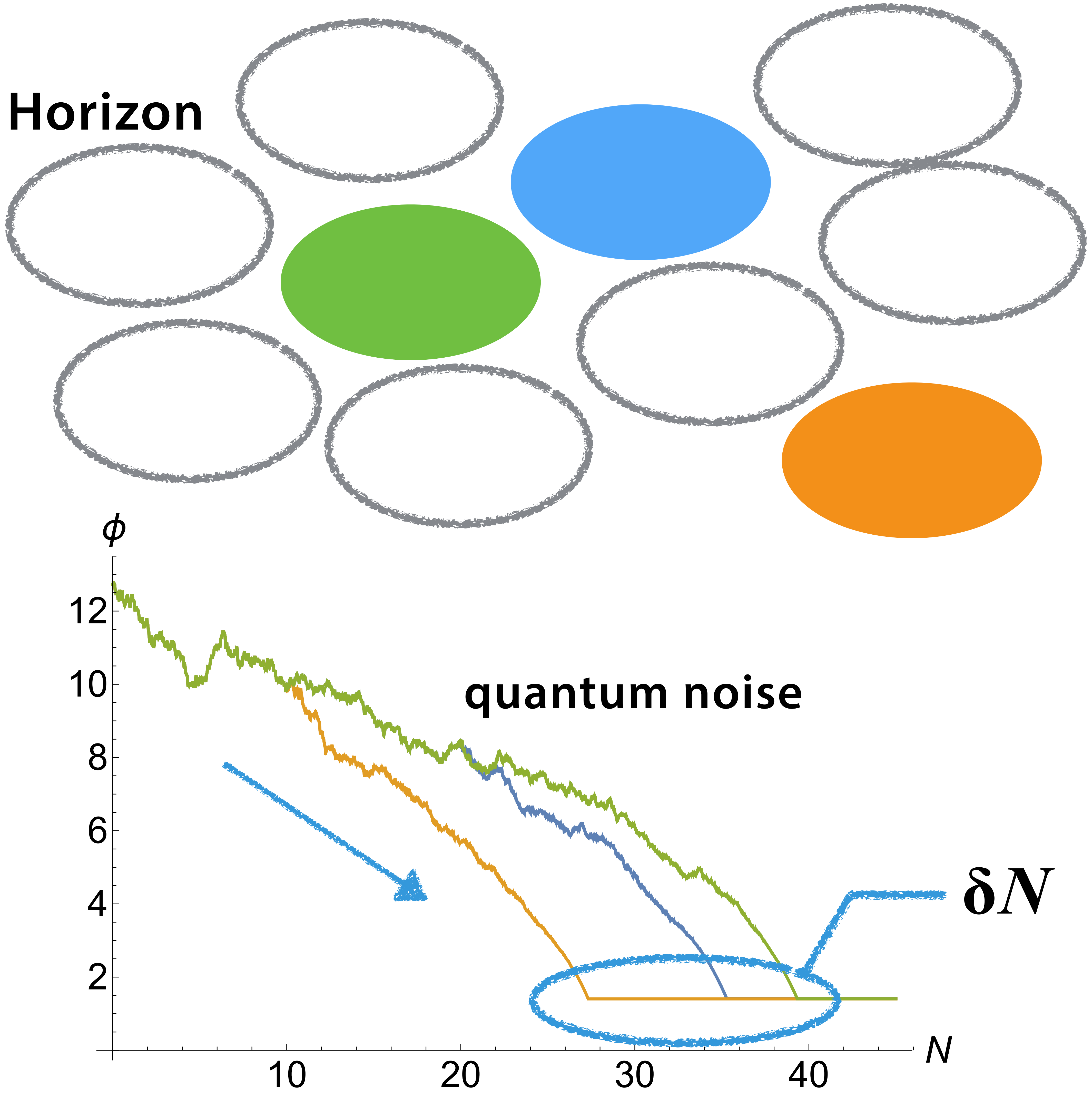
$$\text{e-folds: } dN = H dt$$

**gauge-inv. curvature perturbation**

$$\zeta(\mathbf{x}) = \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N \rangle$$

$$\sigma_\zeta^2 = \langle (N(\mathbf{x}) - \langle N \rangle)^2 \rangle$$

in non-perturbative way



# before Parameter Search

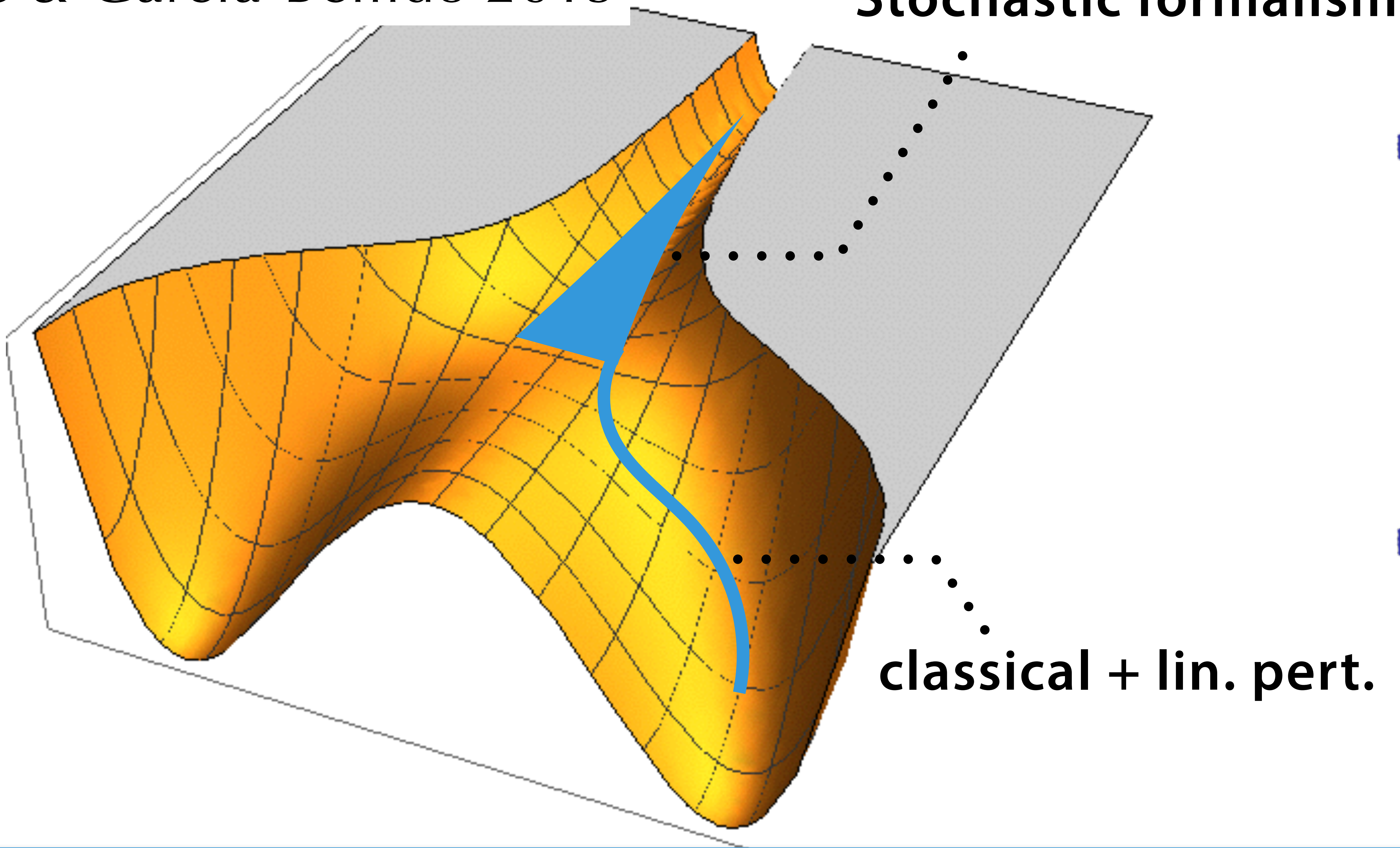
$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2+2\frac{\phi^2\psi^2}{\phi_c^2M^2}\right]+V(\phi)$$

$$V(\phi)\simeq\Lambda^4\frac{\phi-\phi_c}{\mu_1}$$

Taylor expansion in mild case

**There are still 4 parameters ($\Lambda$, $M$, $\phi_c$, $\mu_1$) …**

Clesse & Garcia-Bellido 2015




- Both of $N_{\rm water}$ and $\mathcal{P}_\zeta$ are determined almost only by the combination $\Pi^2 := M^2\phi_c\mu_1/M_p{}^4$
- The peak of $\mathcal{P}_\zeta$ is @ $\phi_c$

# Parameter Search

Kawasaki, YT 2015

$$V(\phi,\psi) = \Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} + \frac{\phi-\phi_c}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2}\right]$$

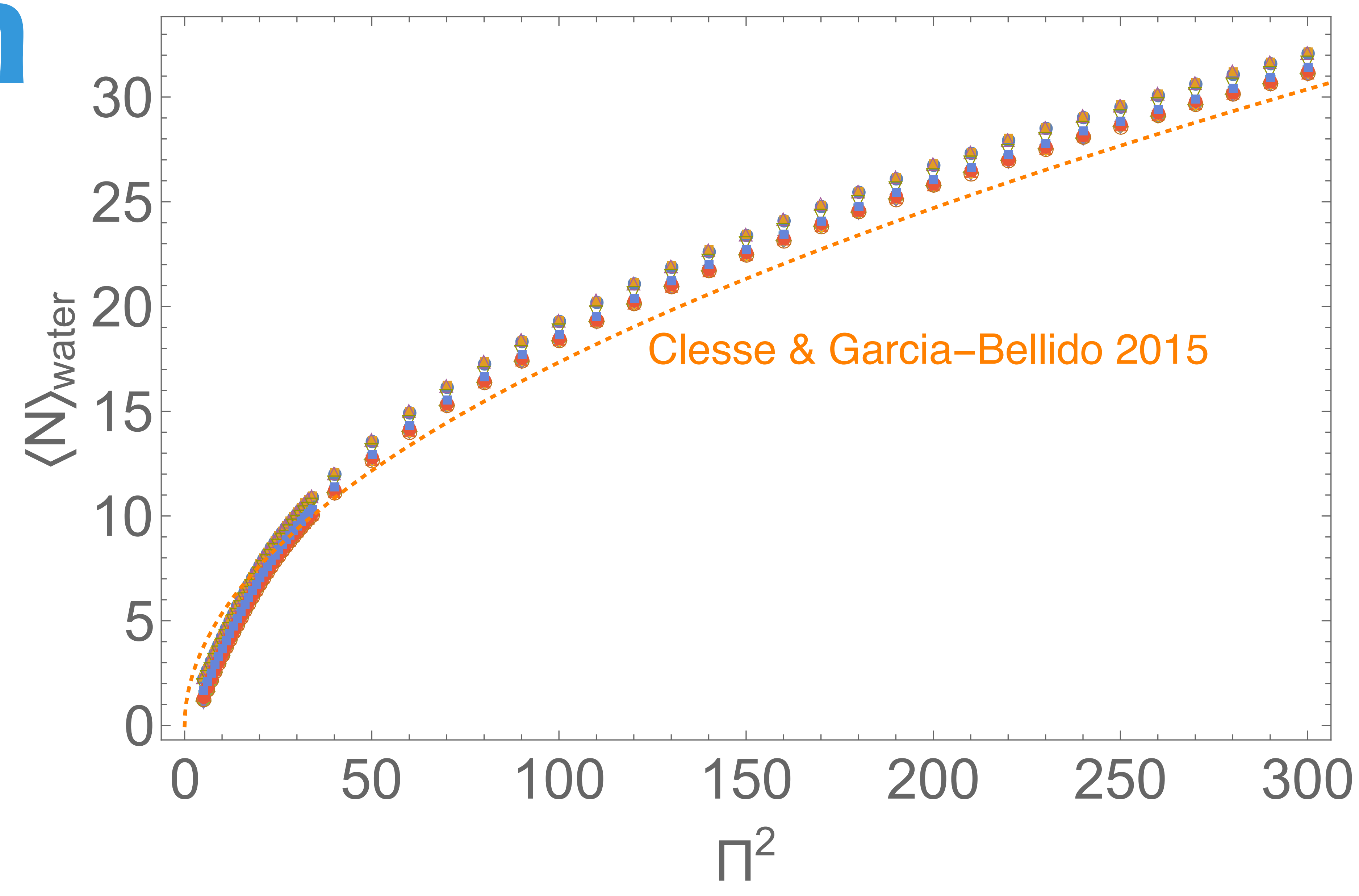
$$\begin{cases} \dfrac{\mu_2}{M_p} = 11, & n_s = 0.9655 \\ \left(\dfrac{\Lambda}{M_p}\right)^4 = 2.198\times10^{-9}\times 12\pi^2\left(\dfrac{\mu_1}{M_p}\right)^{-2}, & A_s = 2.198\times 10^{-9} \end{cases}$$

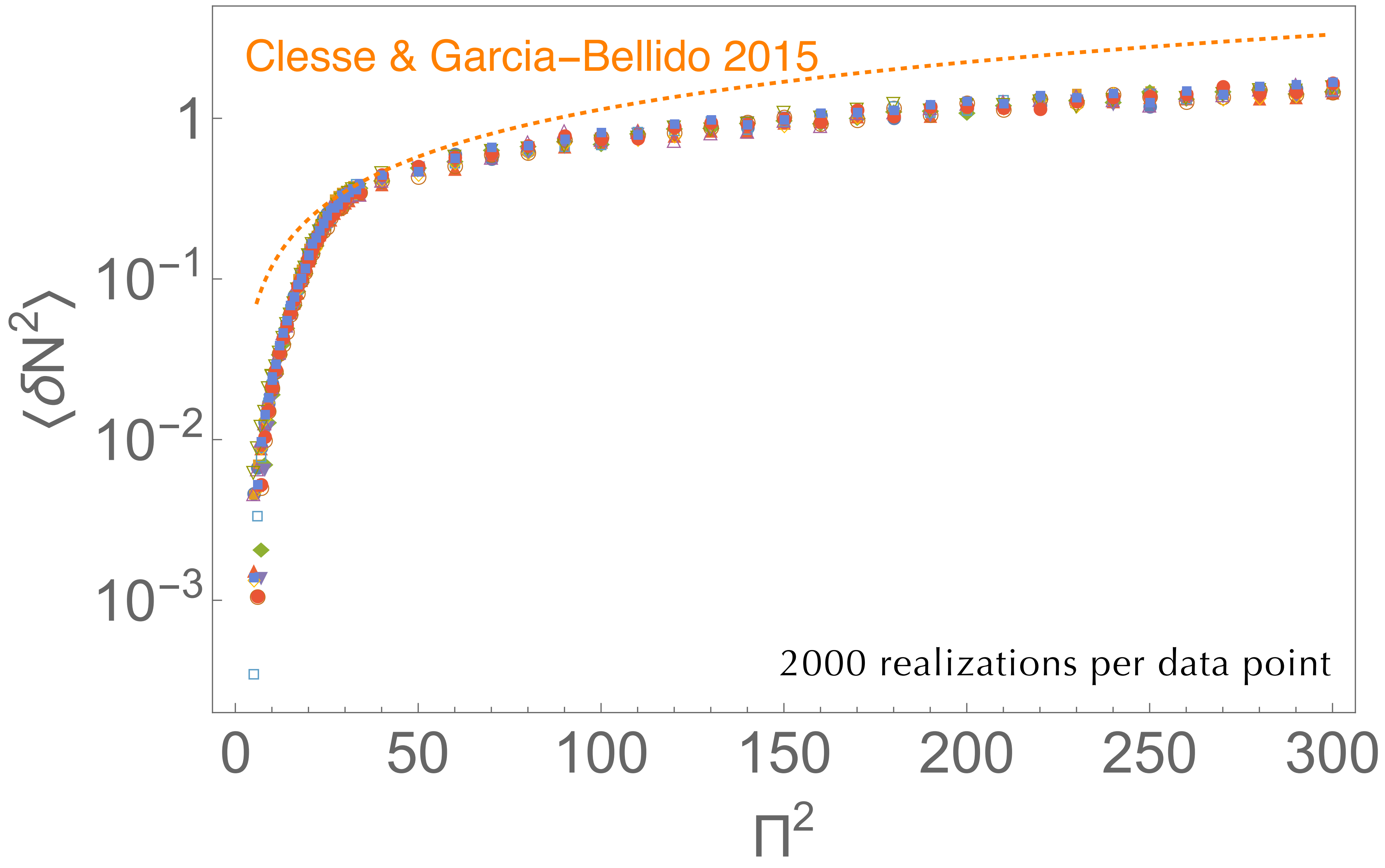
Planck 2015

**Searching region**

$$\begin{cases} \bullet\ \phi_c = \sqrt{2}M\ (\text{SUSY like}), & 10^{-4}M_p \le M \le 10^{-1}M_p \\ \bullet\ \phi_c = 0.1M_p\ (\text{fixed}), & 10^{-4}M_p \le M \le 10^{-1}M_p \\ \bullet\ M = 0.1M_p\ (\text{fixed}), & 10^{-4}M_p \le \phi_c \le 10^{-1}M_p \end{cases}$$

$\mu_1$ is also varied among $0 \lesssim \Pi^2 = \dfrac{M^2\phi_c\mu_1}{M_p^4} \lesssim 300$



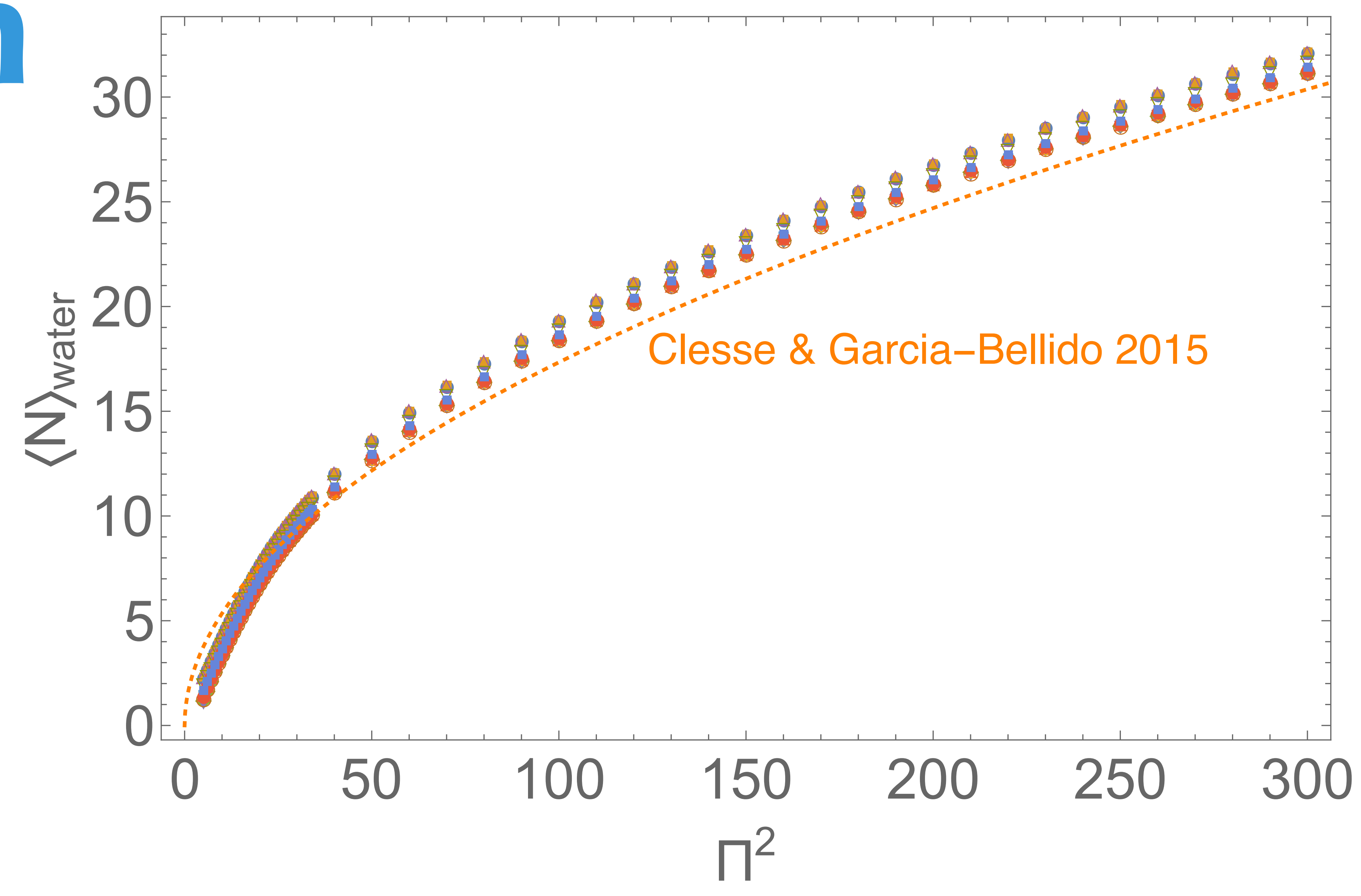



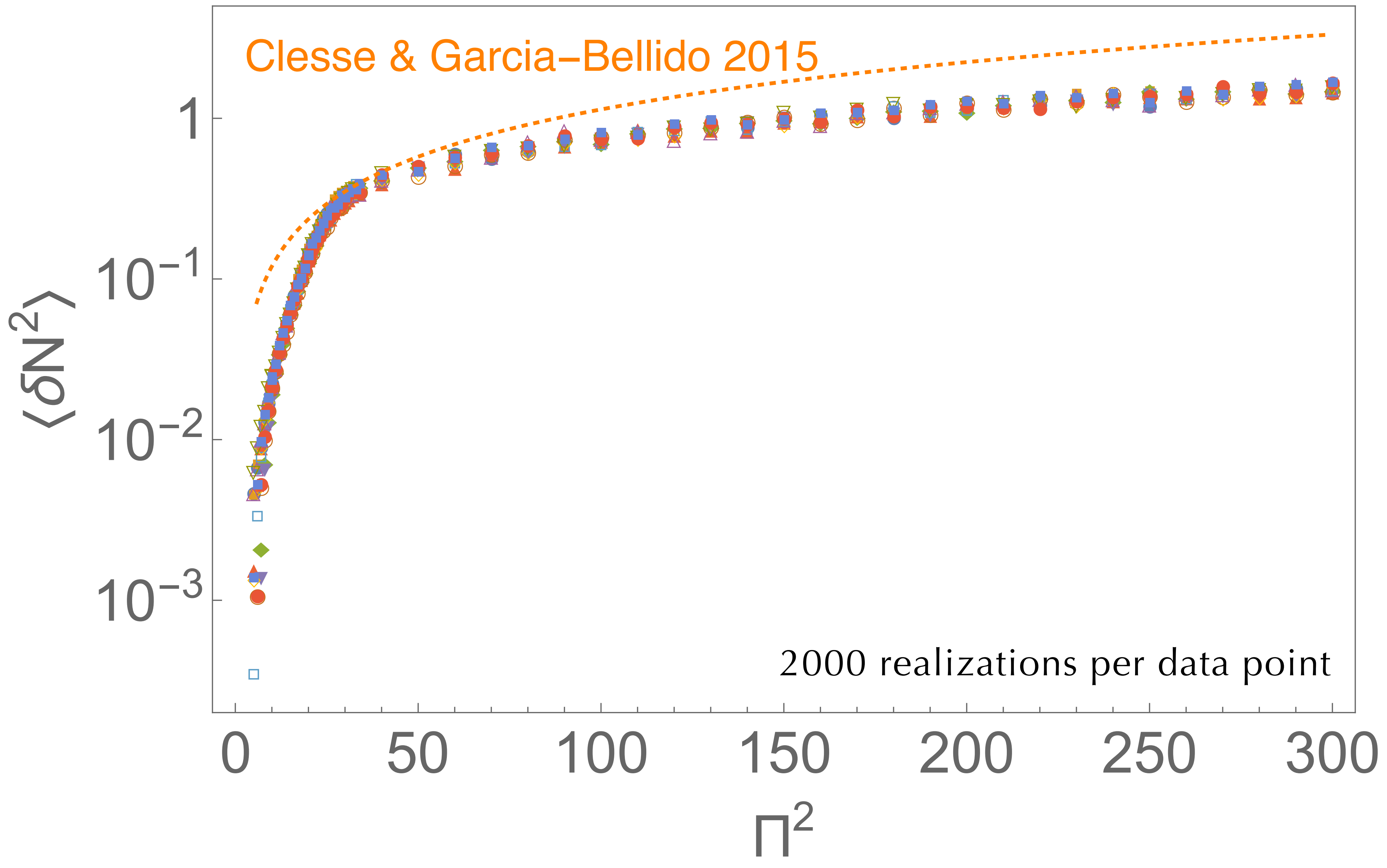
# Parameter Search

- Indeed $\Pi^2$ plays key role beyond the pert. th.
- There are factor differences @ $\langle \delta N^2 \rangle$
- $\langle \delta N^2 \rangle \lesssim 0.01$ corresponds with $\Pi^2 \lesssim 10$.

  → Waterfall phase $\lesssim 5$ e-folds

**Inversely, if waterfall phase $\gtrsim 5$ e-folds,**
**PBHs will be overproduced!!**



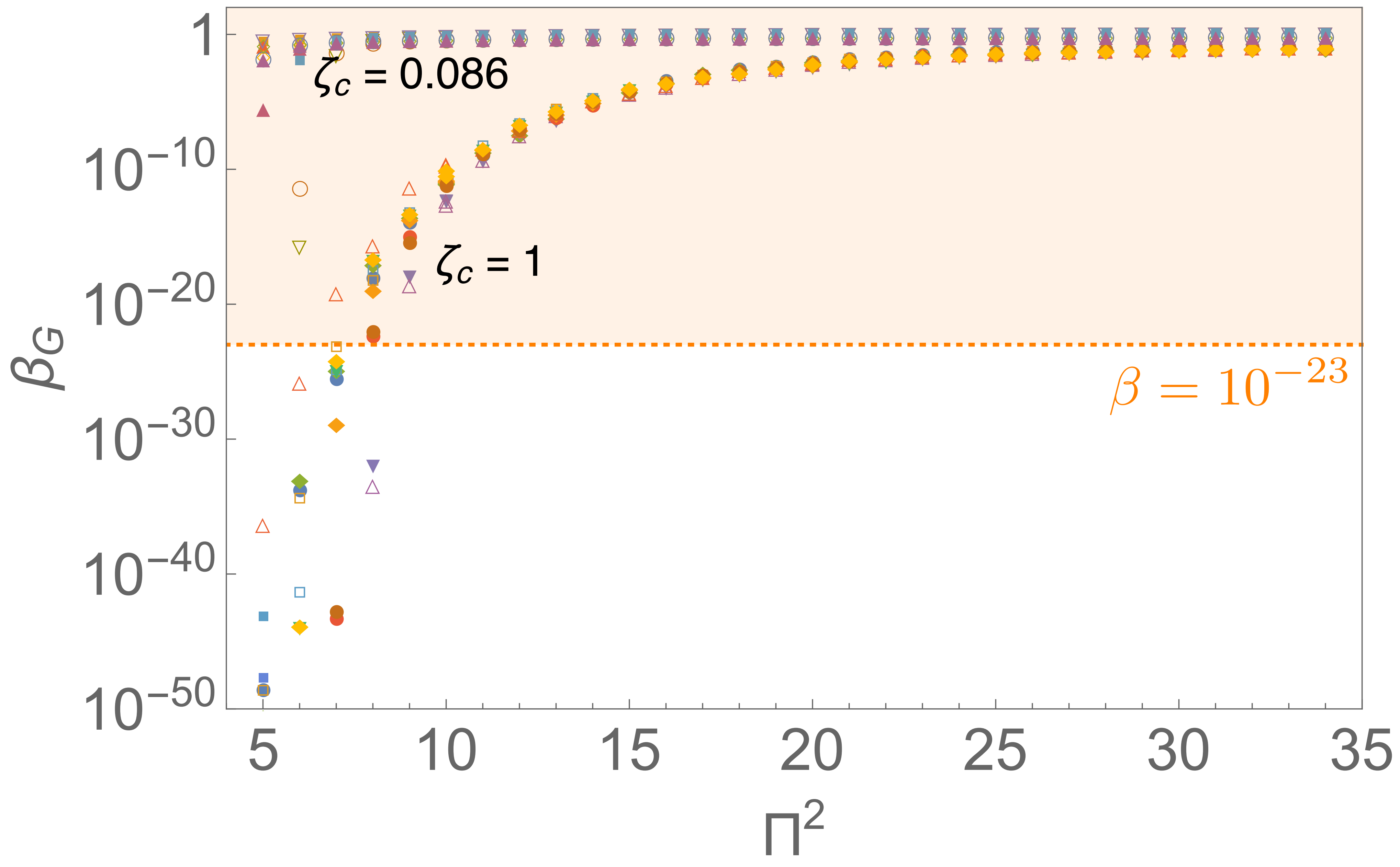
# Precise PBH abundance

PBH formation rate

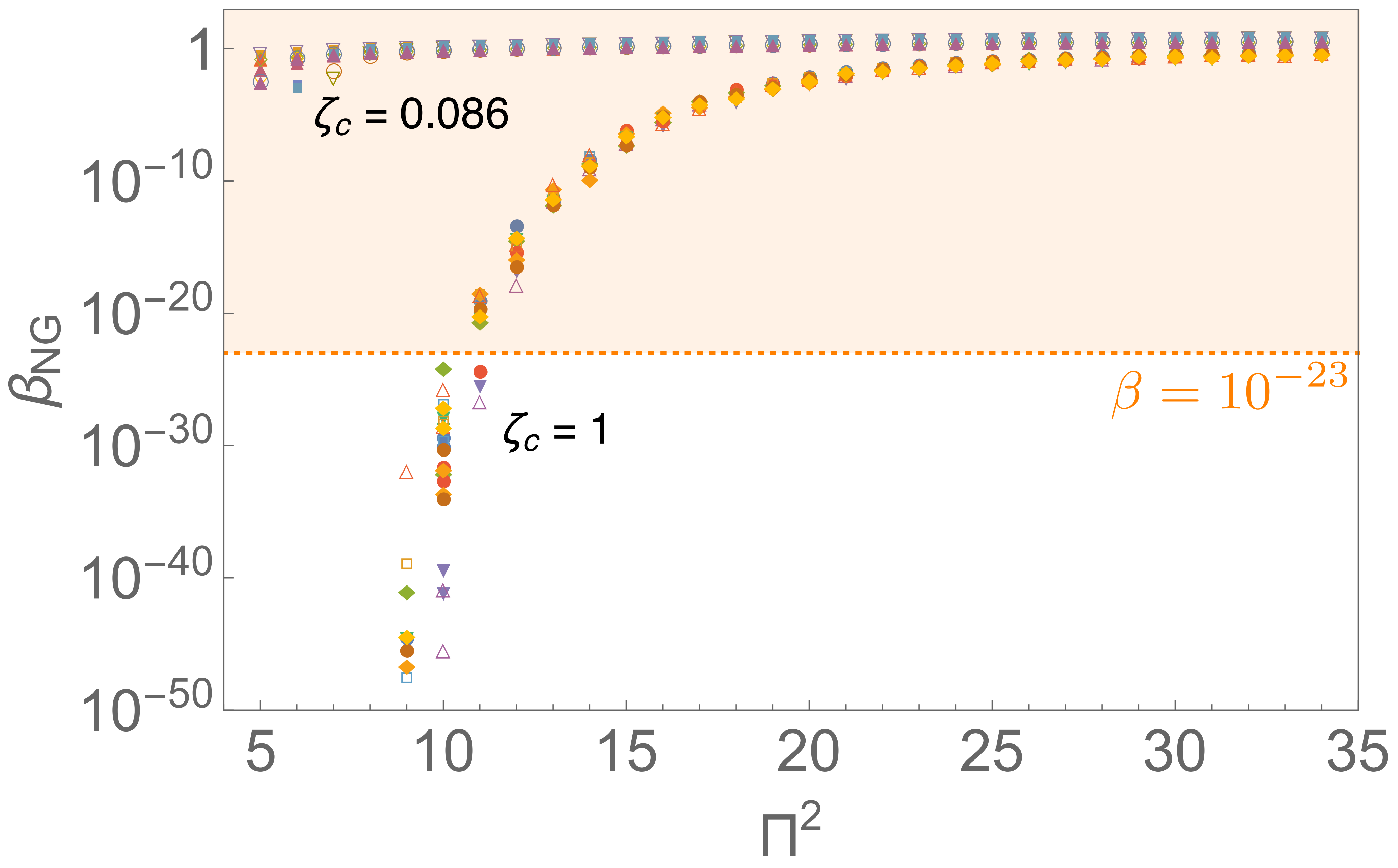
$$\beta = 2\int_{\zeta_c} P(\zeta)\mathrm{d}\zeta \qquad \zeta_c = \begin{cases} 1, \\ \frac{1}{3}\log \left.\frac{3(\chi_a - \sin\chi_a\cos\chi_a)}{2\sin^3\chi_a}\right|_{\chi=\pi\sqrt{\omega}/(1+3\omega)|_{\omega=1/3}} \simeq 0.086. \end{cases}$$

Harada, Yoo, Kohri 2013

- $P(\zeta) = \frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{-\frac{\zeta^2}{2\sigma^2}}, \quad \sigma^2 = \langle\delta N^2\rangle$

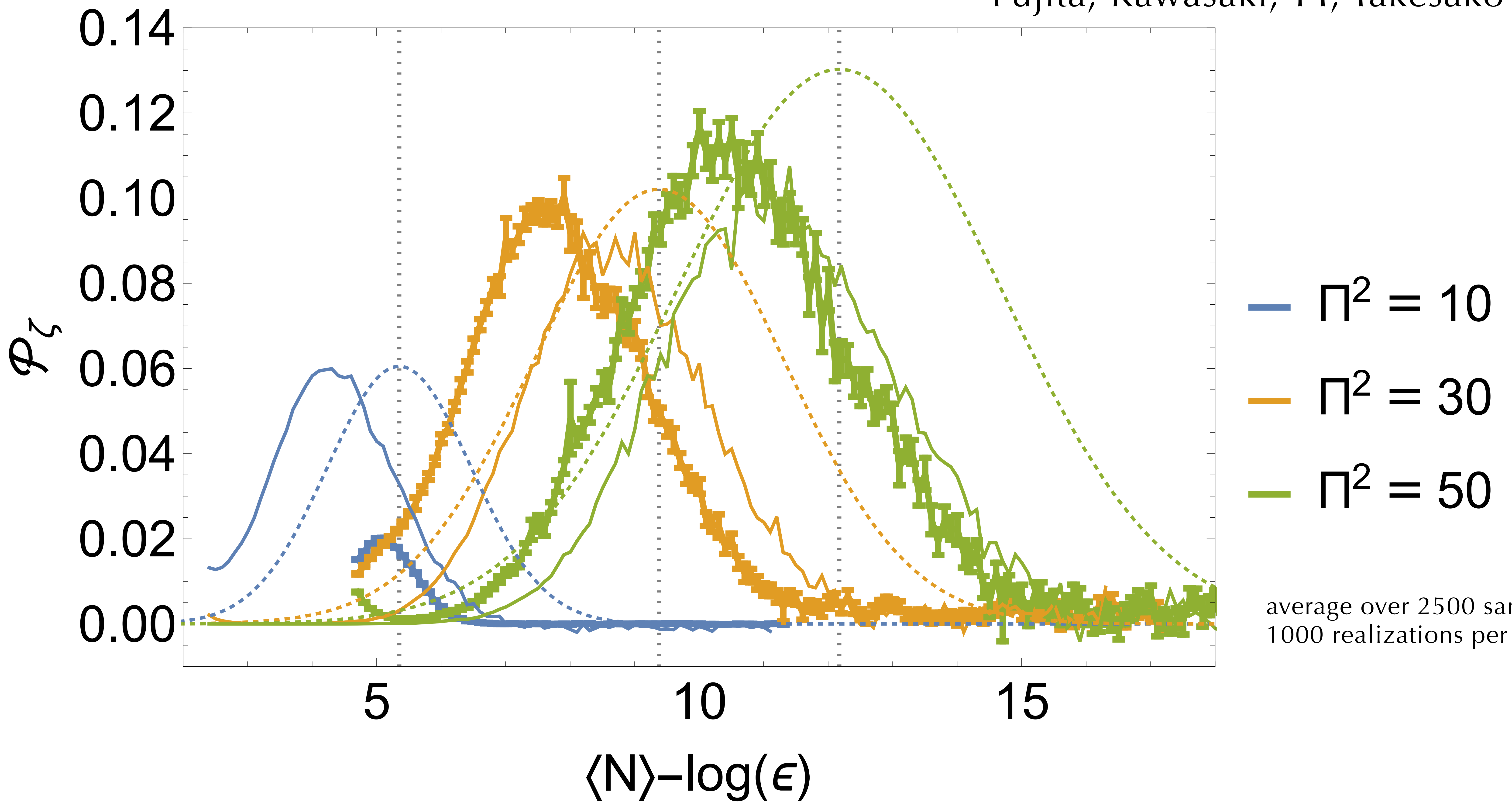



- $P(\zeta) = \exp\left[\sum_{n=3}\frac{(-1)^n}{n!}\langle\delta N^n\rangle_c \frac{\partial^n}{\partial\zeta^n}\right]\exp\left[-\frac{\zeta^2}{2\sigma^2}\right], \quad n \leq 4$



# corresponding PBH scale

Fujita, Kawasaki, YT, Takesako 2013




average over 2500 sample paths
1000 realizations per data point

$$\Pi^2 \sim 10 \longleftrightarrow N_{\rm peak} \sim 4 \longleftrightarrow M_{\rm PBH} \sim \frac{M_p^2}{H_{\rm inf}} e^{2\times 4} = 1.5 \times 10^{-26} M_\odot \left(\frac{10^9\,{\rm GeV}}{H_{\rm inf}}\right)$$

# Conclusions

- Stochastic + $\delta N$ formalism → non-perturbative algorithm
- Can massive PBHs be produced in mild waterfall hybrid inflation?

  → Yes, but rather overproduced and such possibilities are excluded.
- If $\zeta_c = 0.086$, there is no parameter region to appropriately produce detectable PBHs.
- If $\zeta_c = 1$, $\Pi^2 = M^2\phi_c\mu_1/M_p^4$ should be less than around 11 ($N_{water} \lesssim 4$), but even in that case, PBHs are too small.

# Appendices

✱ Clesse & Garcia-Bellido 2015

- I.C. of $\psi$ @ critical point

$$\sigma_\psi^2 = \langle \psi^2 \rangle |_{\phi=\phi_c} = \frac{\sqrt{2}\Lambda^4 M \phi_c^{1/2} \mu_1^{1/2}}{96\pi^{3/2} M_p^4}$$

- EoM

$$\begin{cases} 3H\dot{\phi} = -V_\phi \simeq -\dfrac{\Lambda^4}{\mu_1} - \dfrac{4\Lambda^4\psi^2}{M^2\phi_c^2}\phi \\ 3H\dot{\psi} = -V_\psi \simeq -\dfrac{4\Lambda^4}{M^2}\left(\dfrac{\phi^2}{\phi_c^2} - 1\right) \end{cases}$$

- E-folds
  - phase 1 $\quad \dfrac{\Lambda^4}{\mu_1} \gg \left|\dfrac{4\Lambda^4\psi^2}{M^2\phi_c^2}\right|$

$$N_1 \simeq \frac{\sqrt{\chi_2} M \phi_c^{1/2} \mu_1^{1/2}}{2M_p^2}$$

  - phase 2 $\quad \dfrac{\Lambda^4}{\mu_1} \ll \left|\dfrac{4\Lambda^4\psi^2}{M^2\phi_c^2}\right|$

$$N_2 \simeq \frac{M \phi_c^{1/2} \mu_1^{1/2}}{4M_p^2 \sqrt{\chi_2}}$$

$$\chi_2 = \log\left(\frac{\phi_c^{1/2} M}{2\mu_1^{1/2}\psi_0}\right)$$

$$N_{\text{water}} \propto \frac{M \phi_c^{1/2} \mu_1^{1/2}}{M_p^2} = \Pi$$

- Power spectrum

$$\mathcal{P}_\zeta \simeq \frac{H^2}{(2\pi)^2}(N_\phi^2 + N_\psi^2)\Big|_{aH=k} \simeq \frac{\Lambda^4 M^2 \phi_c \mu_1}{192\pi^2 M_p^6 \chi_2 \psi^2|_{aH=k}}$$

especially $\quad \mathcal{P}_{\zeta,\max} \simeq \dfrac{M\phi_c^{1/2}\mu_1^{1/2}}{2\sqrt{2\pi} M_p^2 \chi_2} \propto \Pi \quad @ \ \phi_c$

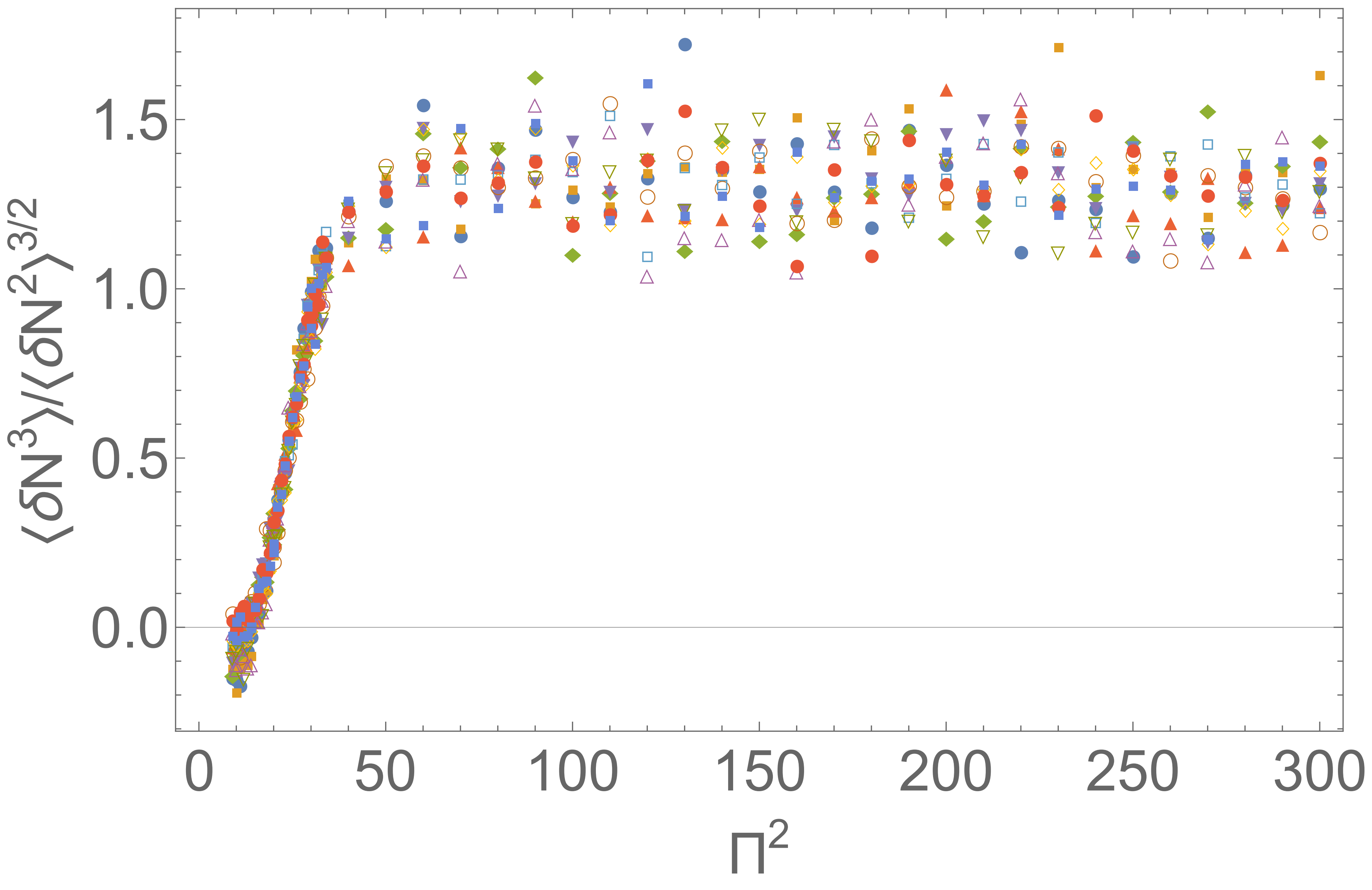
# Appendices

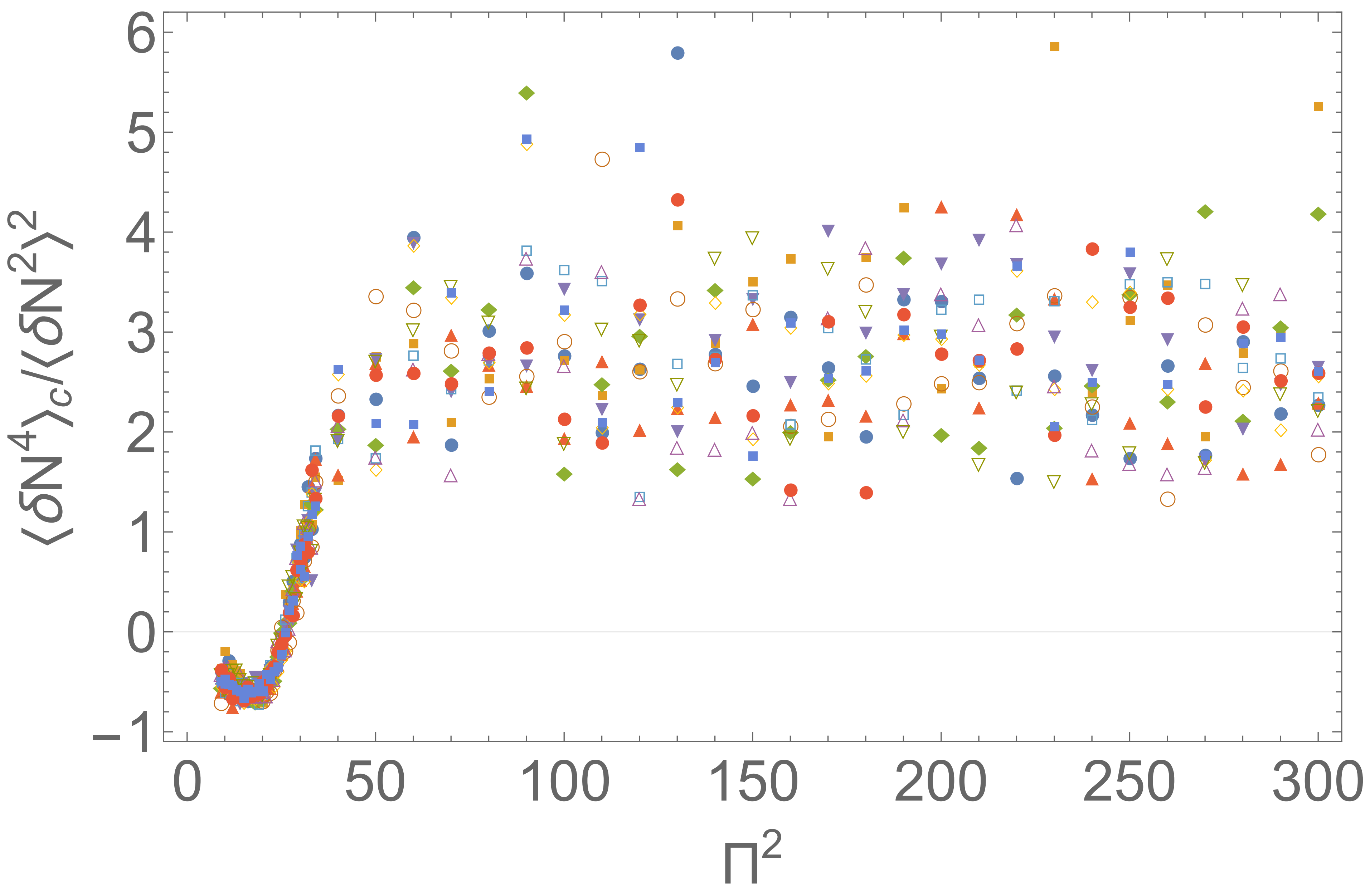
- Full EoM

$$
\begin{cases}
\dfrac{\mathrm{d}\phi_i}{\mathrm{d}N}(N) = \dfrac{\pi_i}{H}(N) + \mathcal{P}_{\phi_i}^{1/2}(N)\xi_i(N), \\
\dfrac{\mathrm{d}\pi_i}{\mathrm{d}N}(N) = -3\pi_i(N) - \dfrac{V_i}{H}(N), \\
V_i(N) = V_i(\phi_1(N), \phi_2(N), \cdots), \\
3M_p^2H^2(N) = \sum_i \dfrac{\pi_i^2}{2} + V(\phi_1(N), \phi_2(N), \cdots), \\
\mathcal{P}_{\phi_i}(N) = \dfrac{H^2}{8\pi}\alpha^3 |H_{\nu_i}^{(1)}(\alpha)|^2, \\
\nu_i = \sqrt{\dfrac{9}{4} - \dfrac{V_{ii}}{H^2}}, \\
\langle \xi_i(N) \rangle = 0, \\
\langle \xi_i(N)\xi_j(N') \rangle = \delta_{ij}\delta(N - N').
\end{cases}
$$

# Appendices

* skewness and kurtosis



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* ε-dependence