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Renormalization-scale uncertainty in the decay rate of false vacuum

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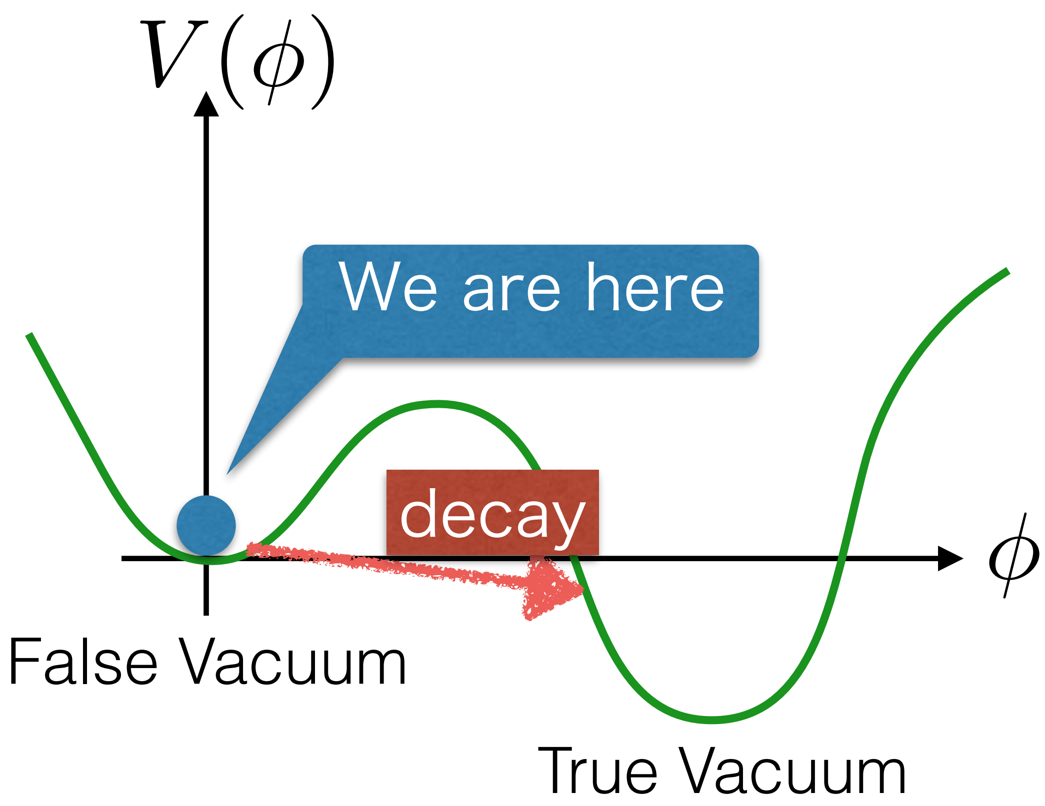
Contents

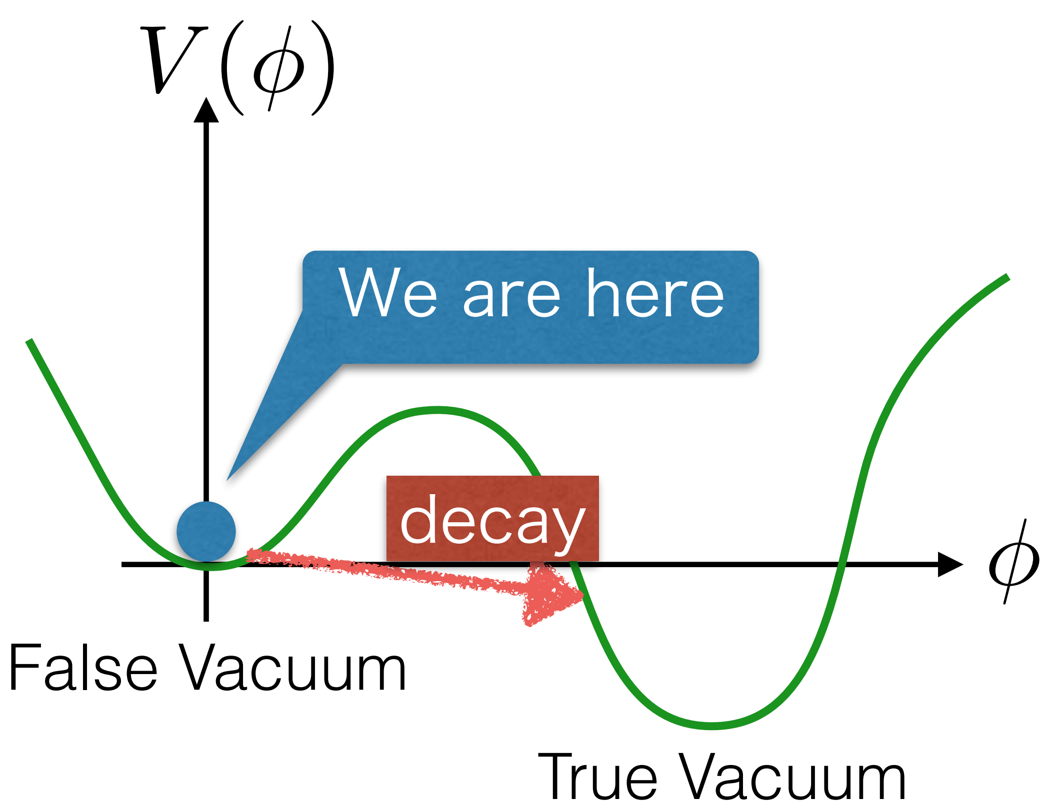
- Introduction
- scale uncertainty and 1-loop calculation
- SM + stau system
- Summary

Introduction

Vacuum decay



Vacuum decay



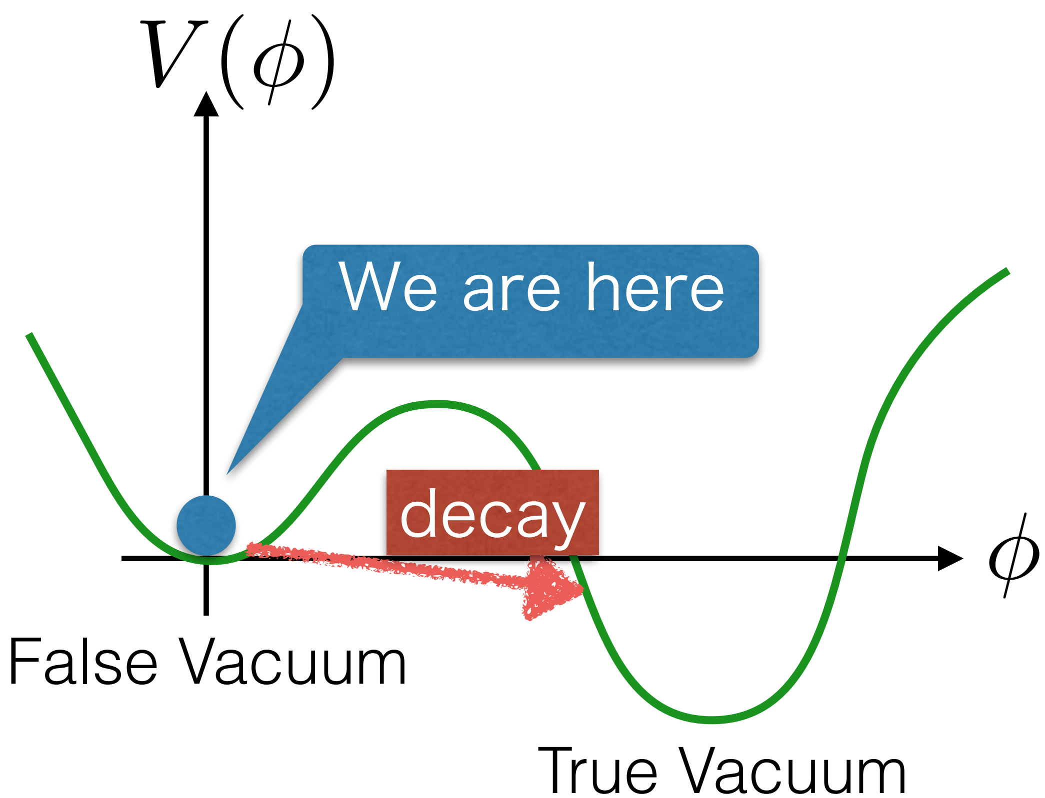
<u>Tree level potential</u>

ex.) Supersymmetry

$h\tilde{t}_L\tilde{t}_R$ Higgs mass, hgg, hγγ, …

$h\tilde{\ell}_L\tilde{\ell}_R$ muon g-2, hγγ, …

Vacuum decay



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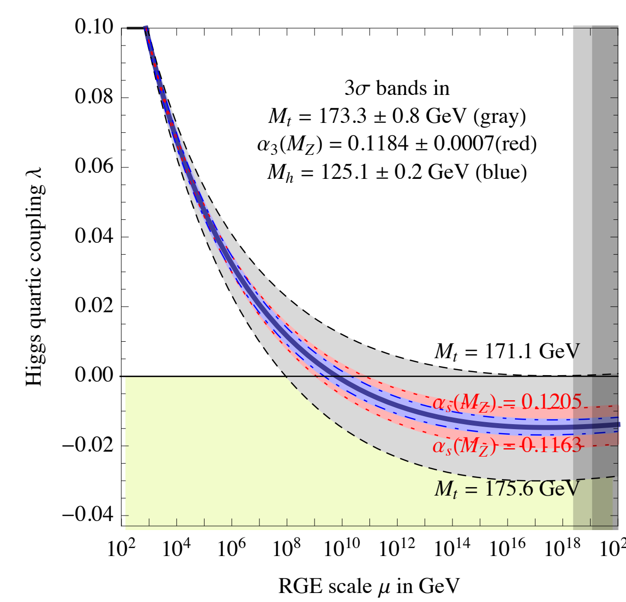
Effective potential

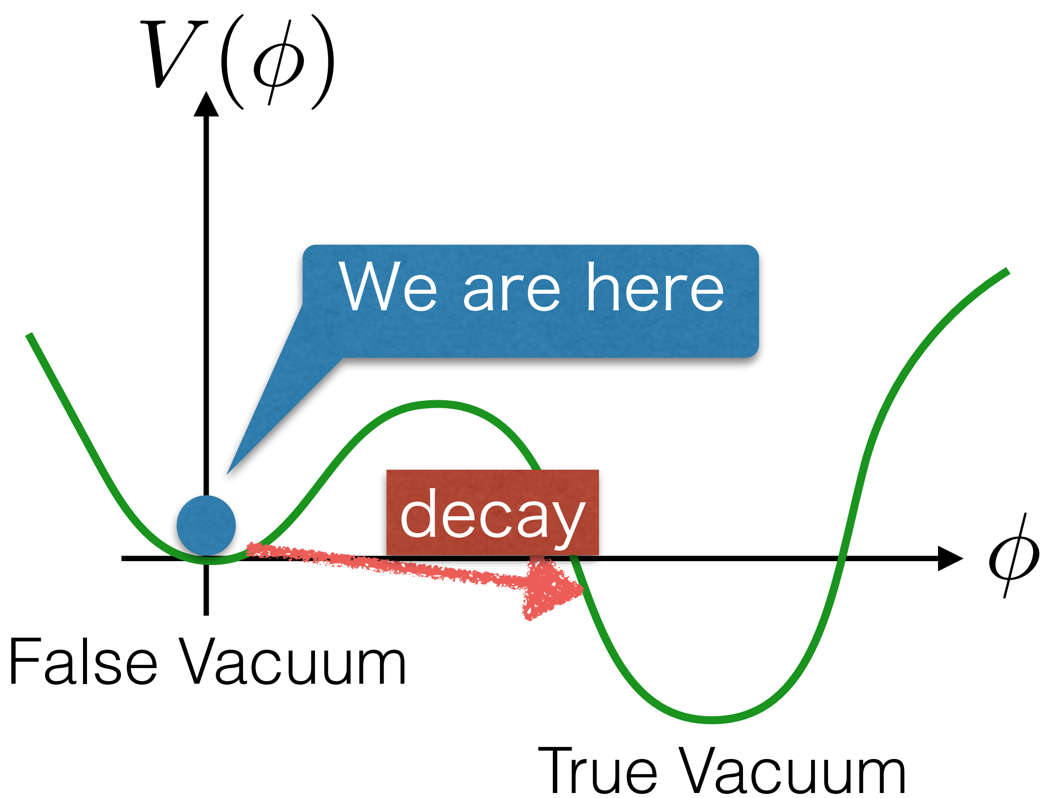
ex.) Standard Model



D. Buttazzo, et.al., 1307.3536/hep-ph

Kyohei's talk

Vacuum decay



Tree level potential

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Effective potential

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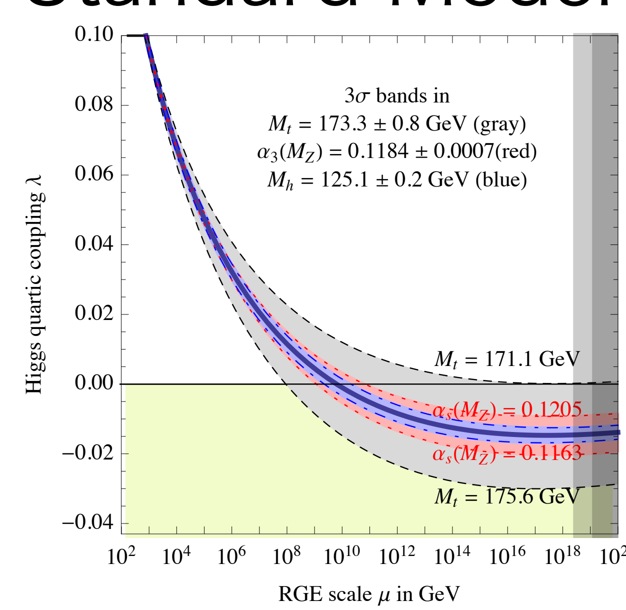

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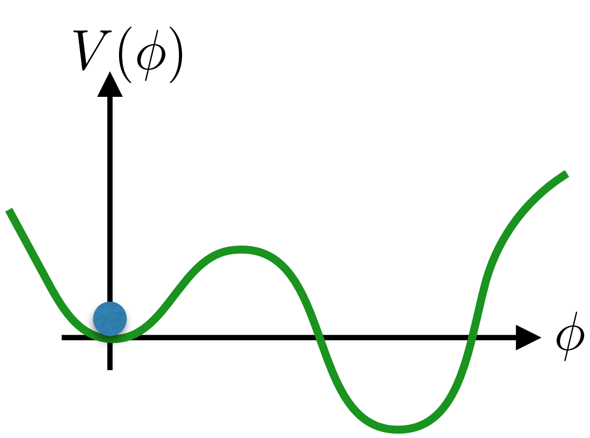
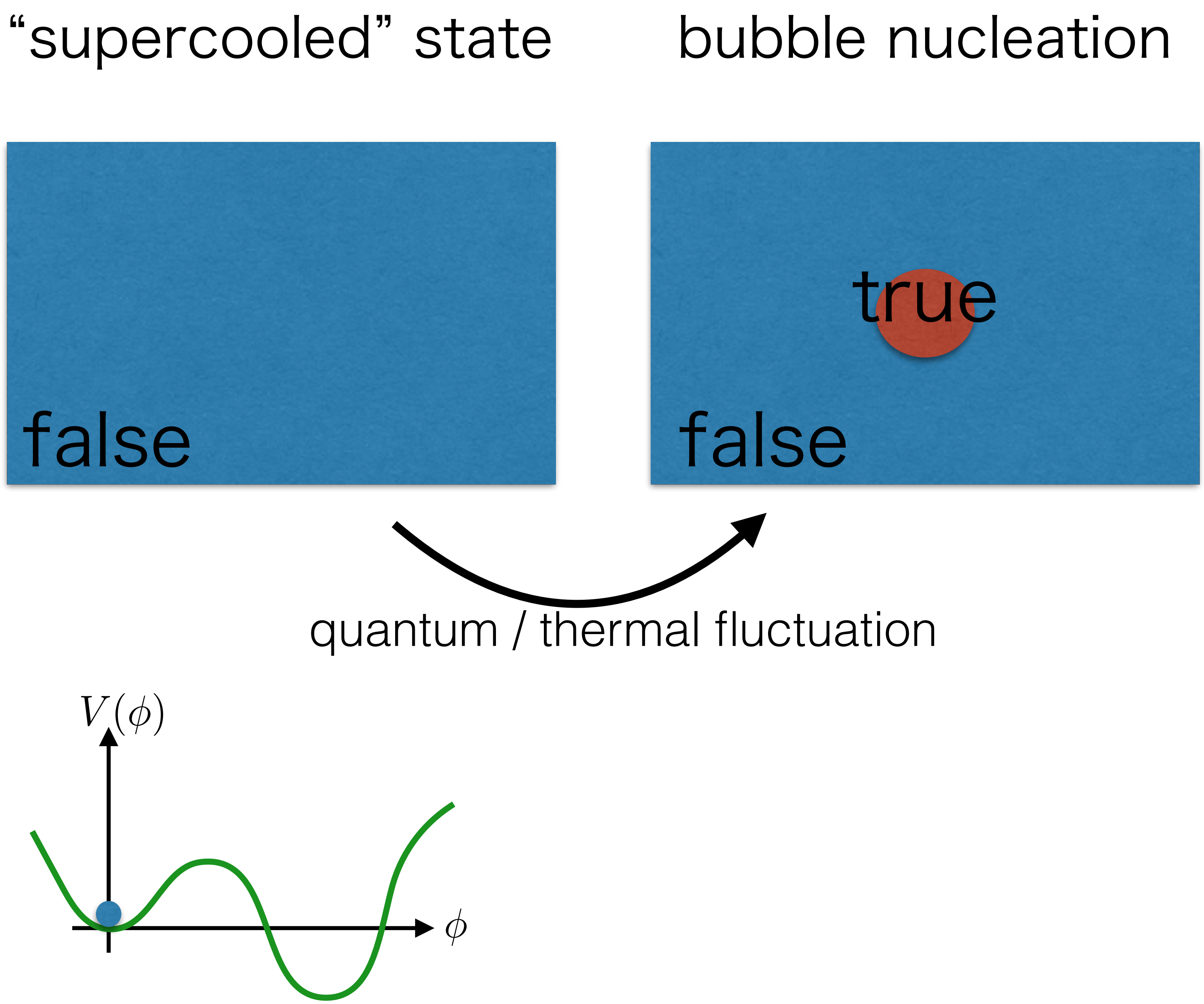
Kyohei's talk

Decay rate

“supercooled” state

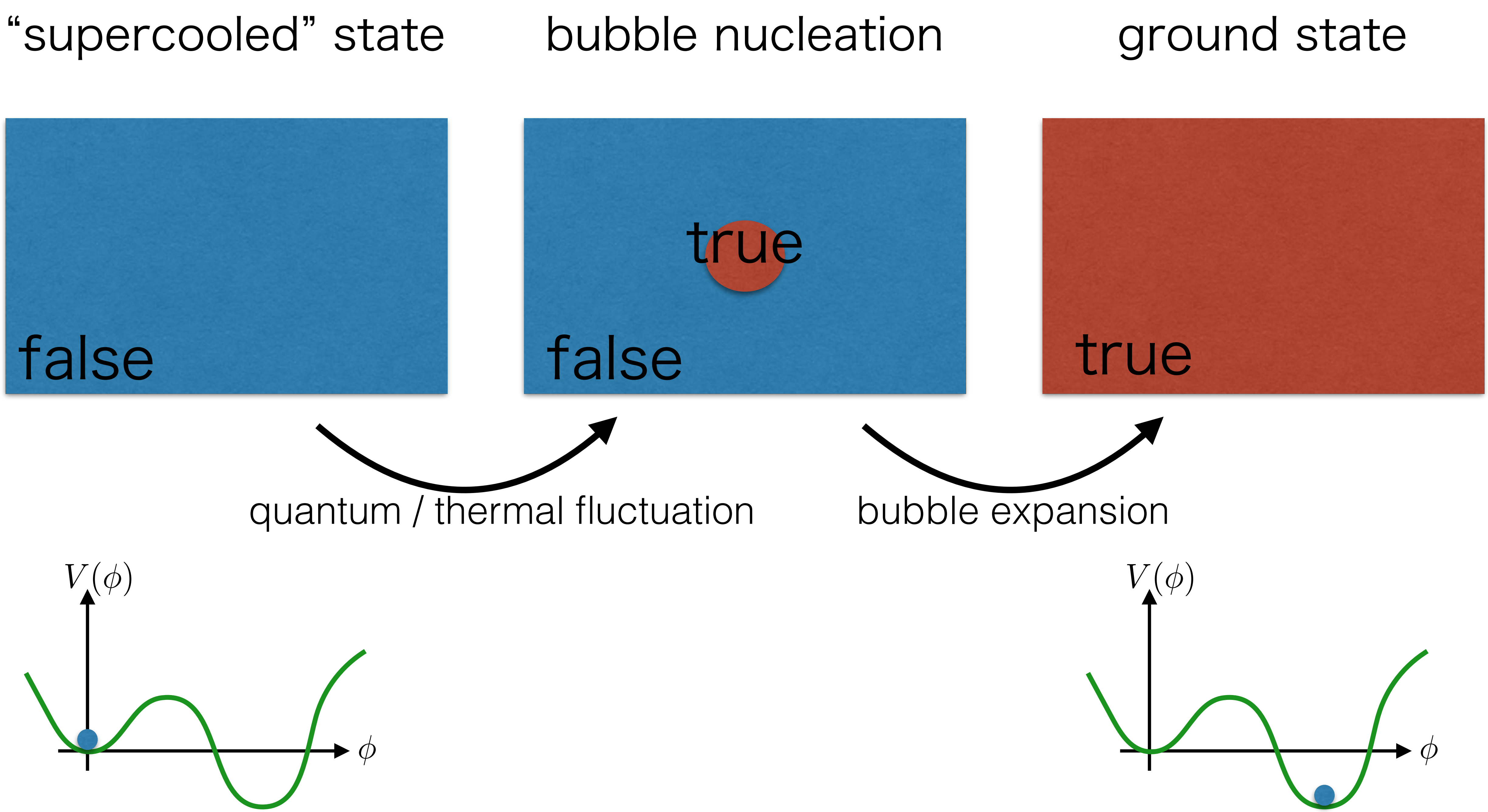


Decay rate



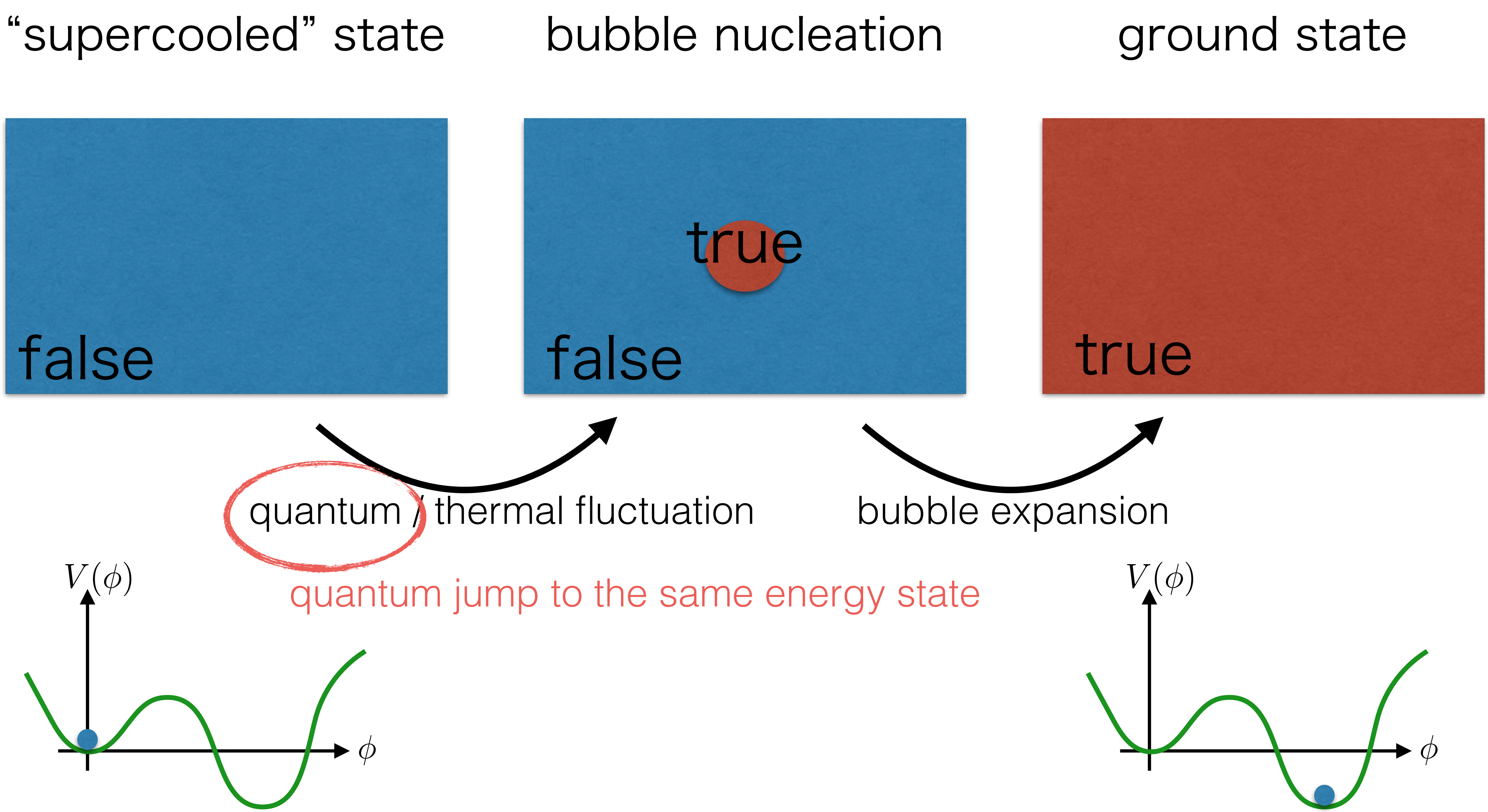
“supercooled” state
bubble nucleation
false
true
false
quantum / thermal fluctuation
$V(\phi)$
ϕ

Decay rate



“supercooled” state
bubble nucleation
ground state
false
true
false
true
quantum / thermal fluctuation
bubble expansion
$V(\phi)$
ϕ
$V(\phi)$
ϕ

Decay rate



"supercooled" state
bubble nucleation
ground state
false
true
false
true
quantum / thermal fluctuation
bubble expansion
quantum jump to the same energy state
$V(\phi)$
ϕ
$V(\phi)$
ϕ

Decay rate

<u>Bubble nucleation rate</u>

$$\gamma = Ae^{-B}$$

“WKB” in QM

bounce action

$$B = S(\phi_B)$$

dim. = 4

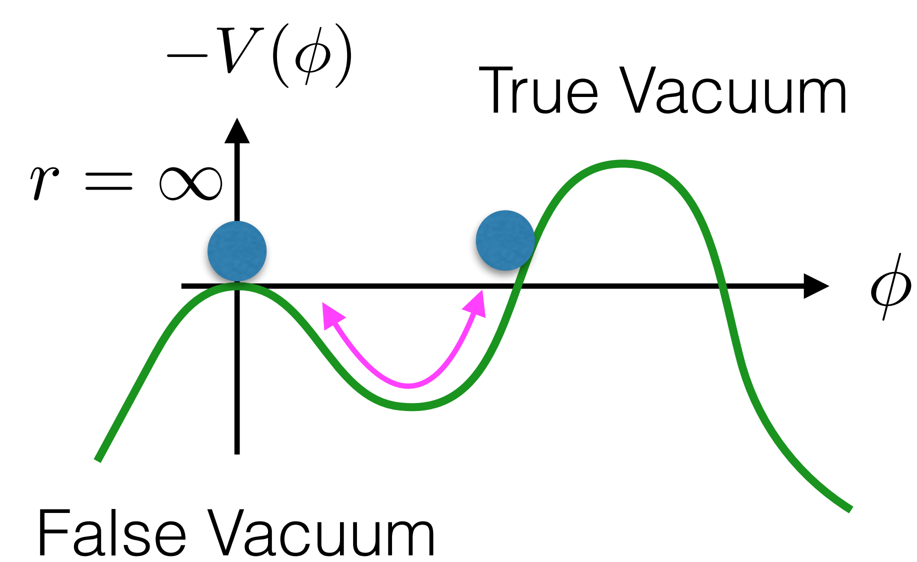
1/(time*volume)

Pre-exponential factor

$$A \sim \mu^4$$

typical scale

bounce solution



O(4) symmetric classical solution

$$\partial^2 \phi = V'(\phi)$$

Toy model

Potential $$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

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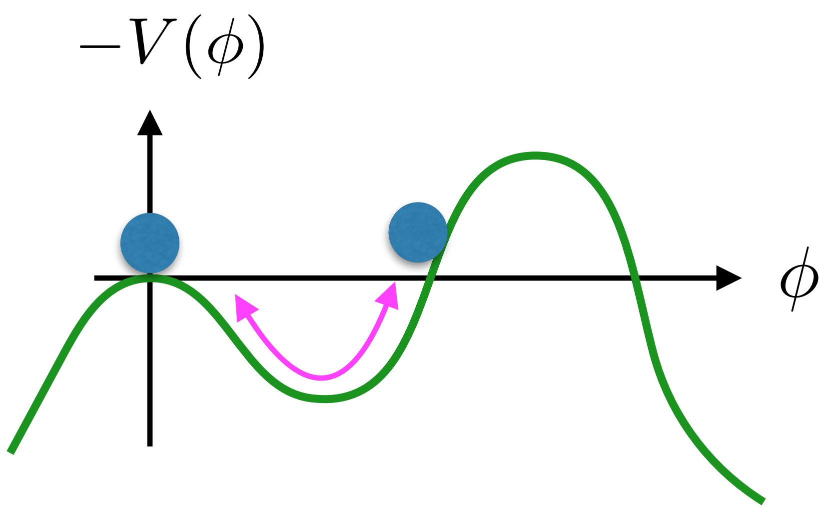
Bounce $$\partial^2\phi = V'(\phi)$$



Toy model

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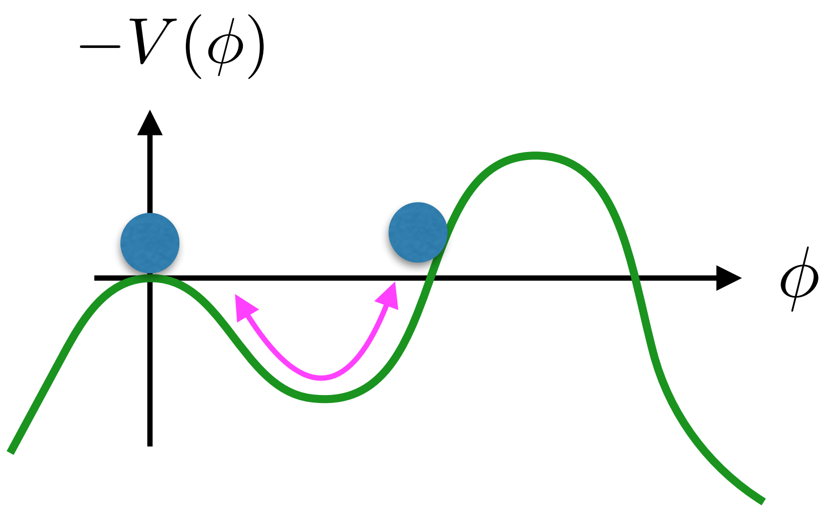


Action $$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B)\right]$$

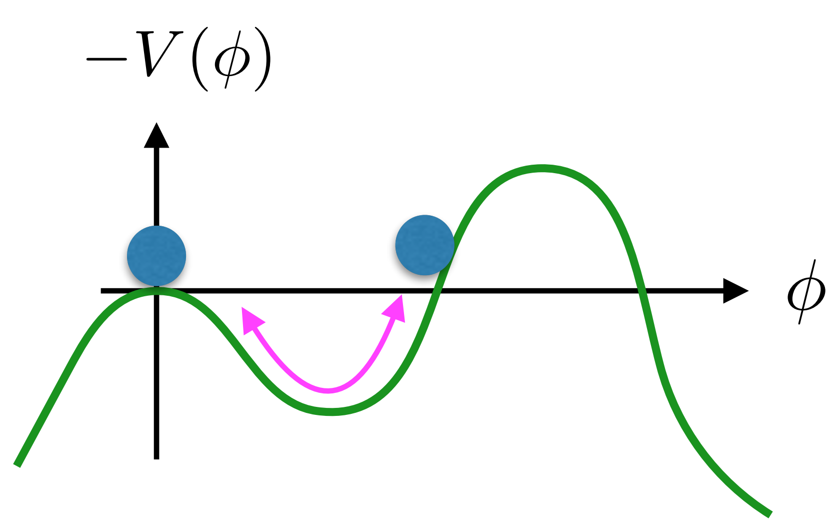
Toy model

Potential

$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

Bounce

$$\partial^2\phi = V'(\phi)$$



Action

$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B)\right]$$

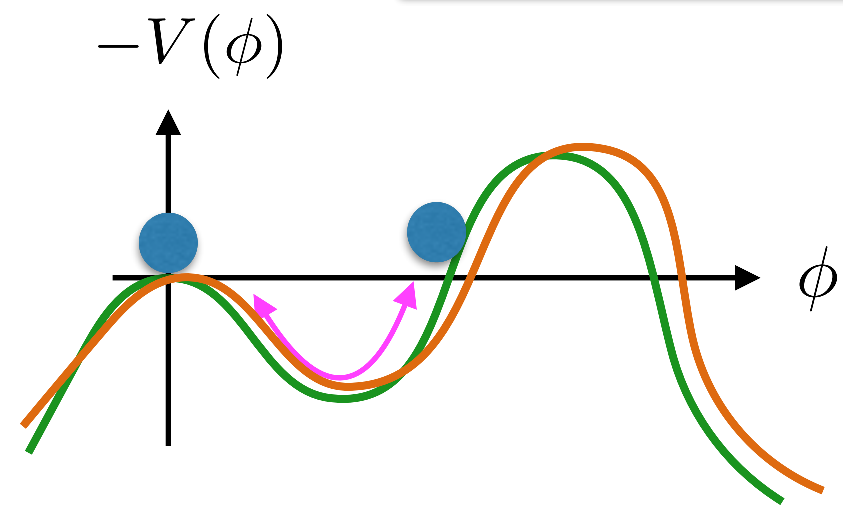
Decay rate

$$\gamma \simeq m^4 e^{-B}$$

Toy model

Potential $$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Scale dependent



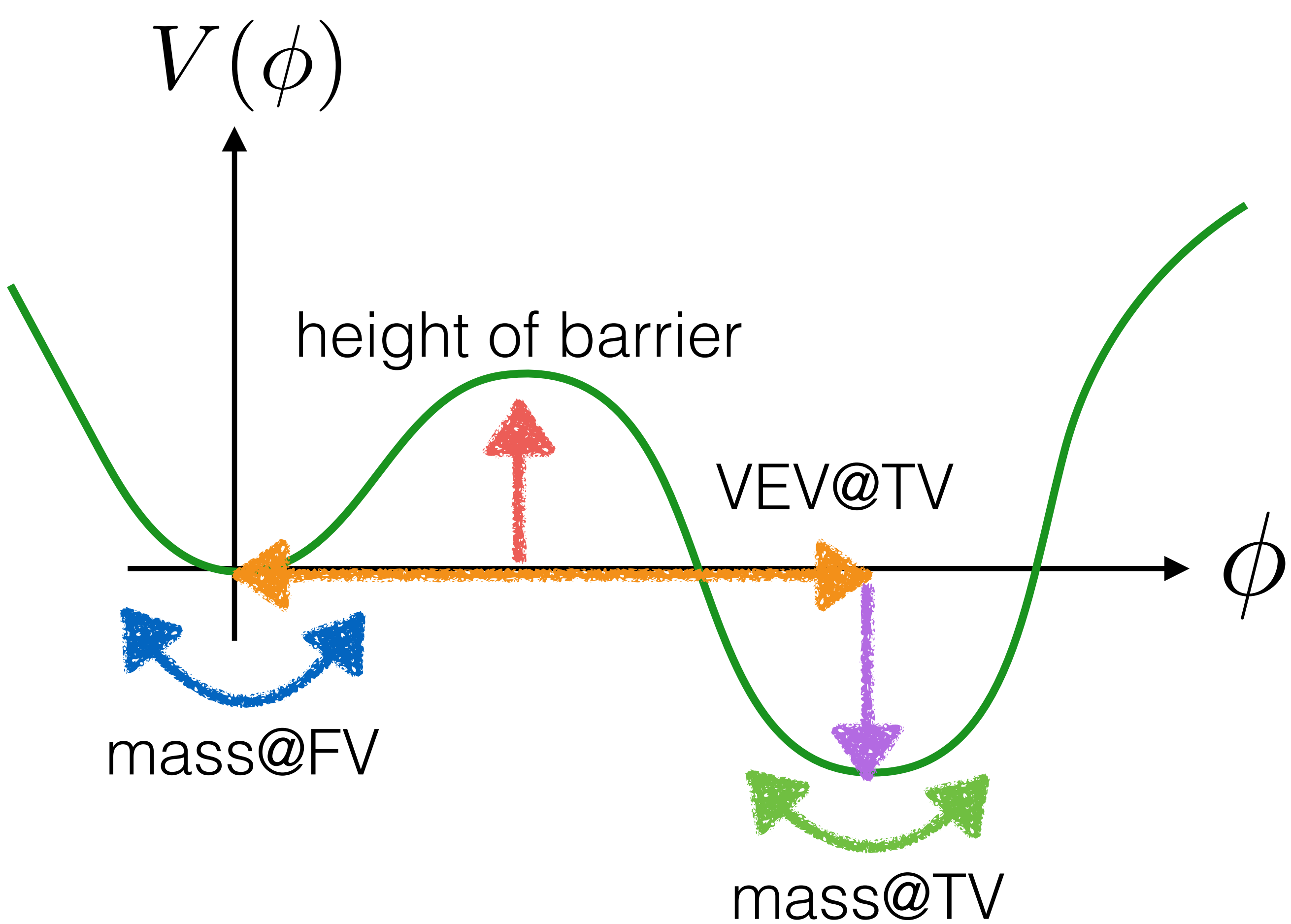
Bounce $$\partial^2\phi = V'(\phi)$$

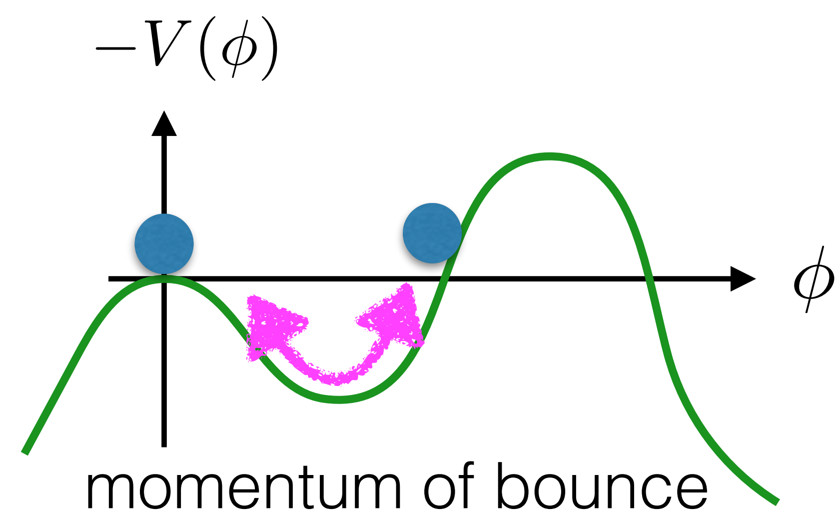
Action $$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B)\right]$$

Decay rate $$\gamma \simeq m^4 e^{-B}$$

Scale?

Renormalization scale





There are too many relevant scales…

How large is the scale dependence?

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Beta functions

$$\beta_t = \frac{3Am^2}{16\pi^2} \qquad \beta_{m^2} = \frac{3}{16\pi^2}(\alpha m^2 + 3A^2)$$

$$\beta_A = \frac{9\alpha A}{16\pi^2} \qquad \beta_\alpha = \frac{9\alpha^2}{16\pi^2}$$

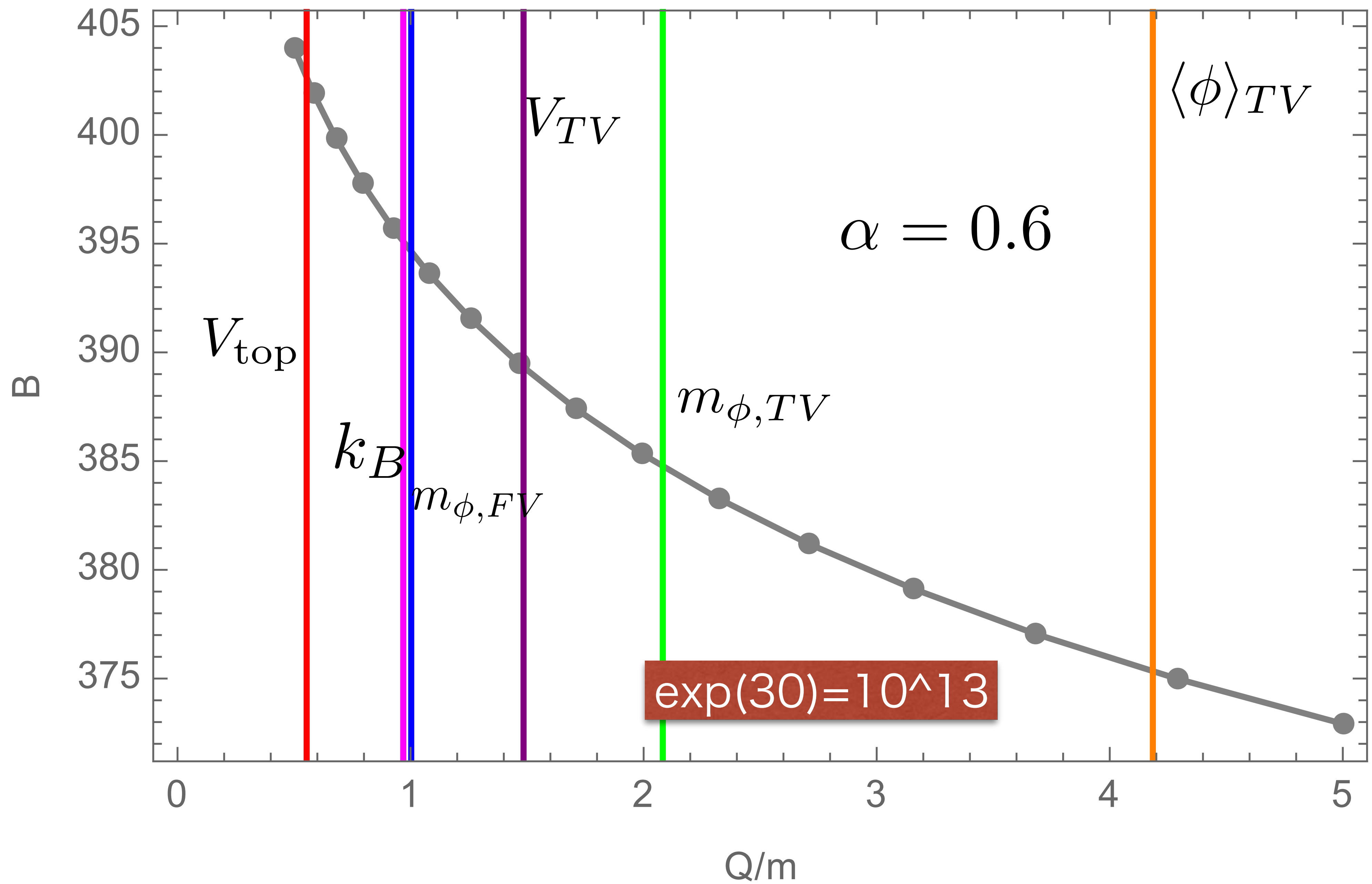
Renormalization conditions

@ $Q = m$

$$\bar{m}^2(m) = m^2,\ \bar{A}(m) = m,\ \bar{t}(m) = 0,\ \bar{\alpha}(m) = \alpha$$

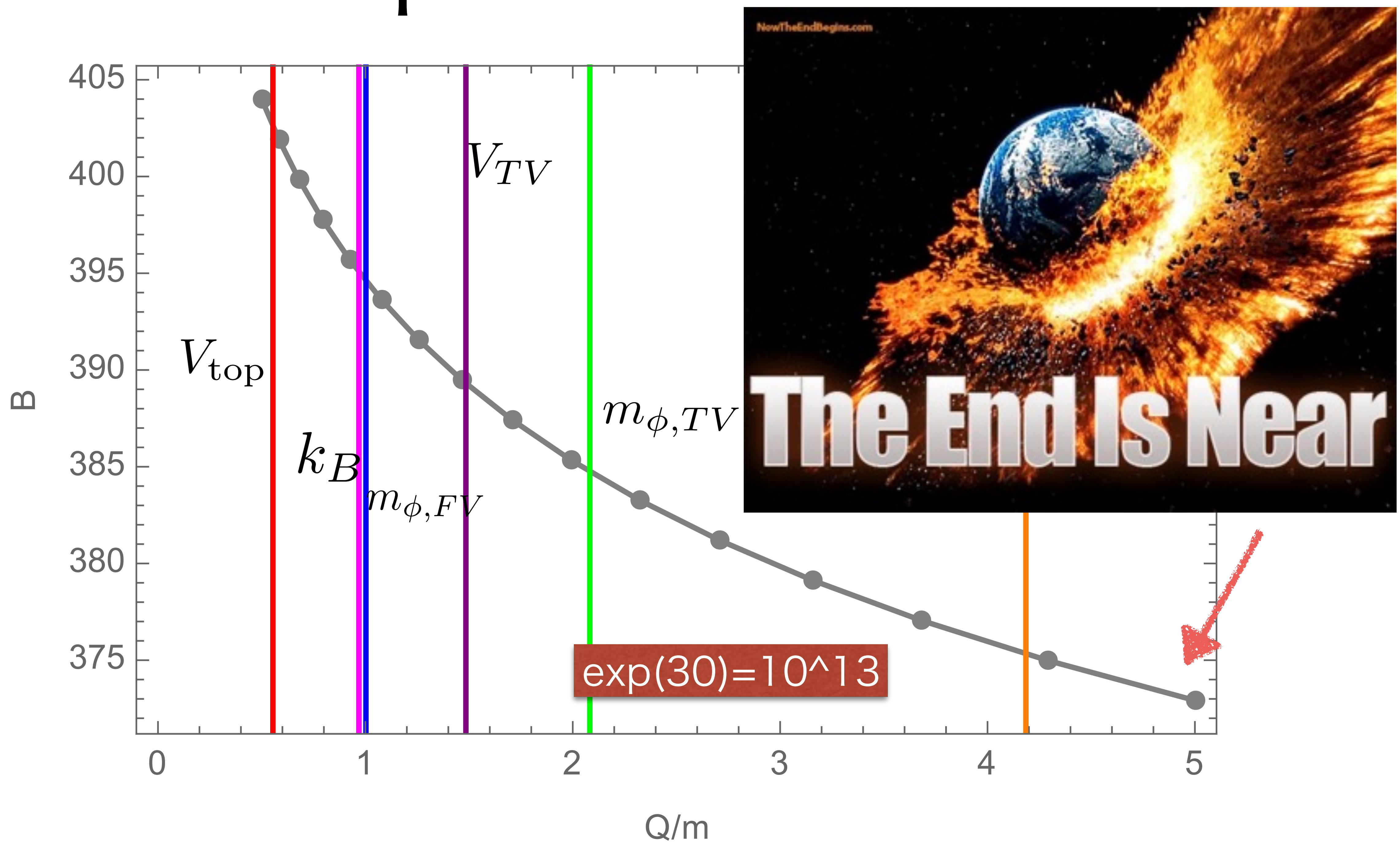
How large is the scale dependence?

CosmoTransitions



can be much larger in a realistic model (top loop)

How large is the scale dependence?



can be much larger in a realistic model (top loop)

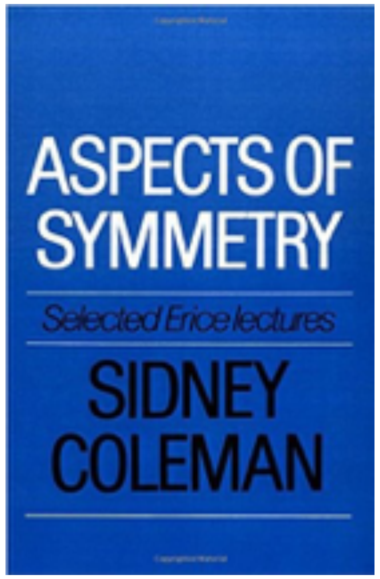
1-loop calculation

Pre-exponential factor

fluctuations around the bounce

$$\gamma = Ae^{-B}$$

$$A = \frac{B^2}{4\pi^2}\left(\frac{\det' S''|_{\mathrm{Bounce}}}{\det S''|_{\mathrm{False}}}\right)^{-1/2}$$



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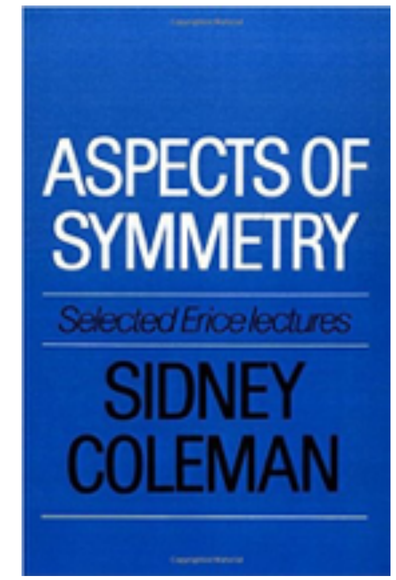
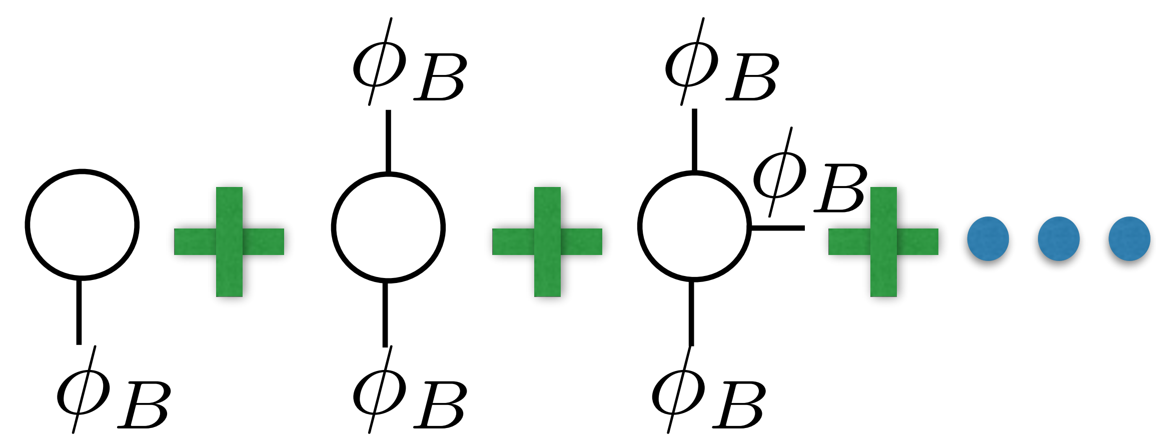


$\simeq$



Pre-exponential factor

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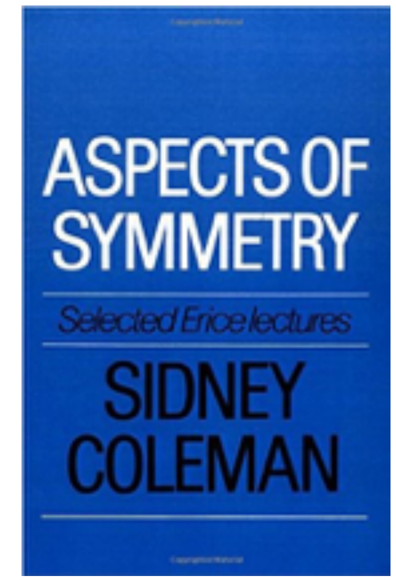
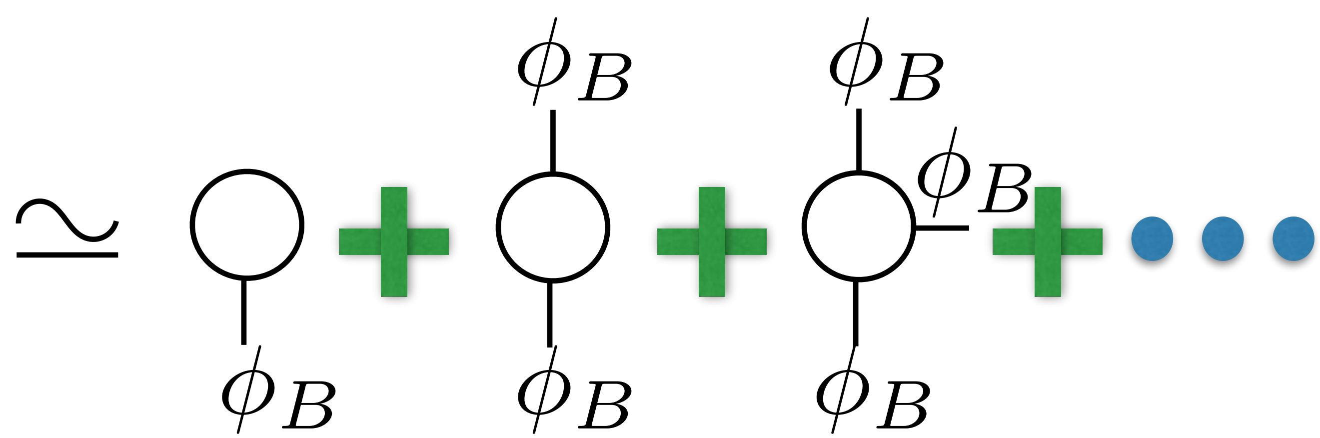
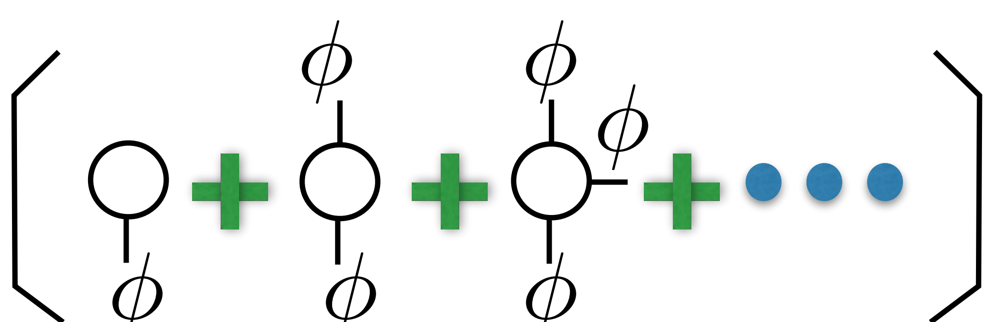




cf.) RGE is related to



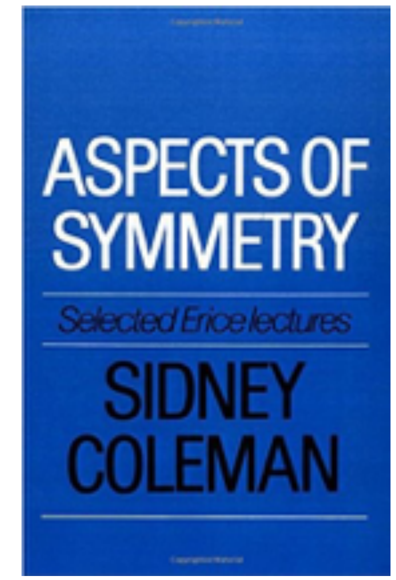
divergent part

Pre-exponential factor

fluctuations around the bounce

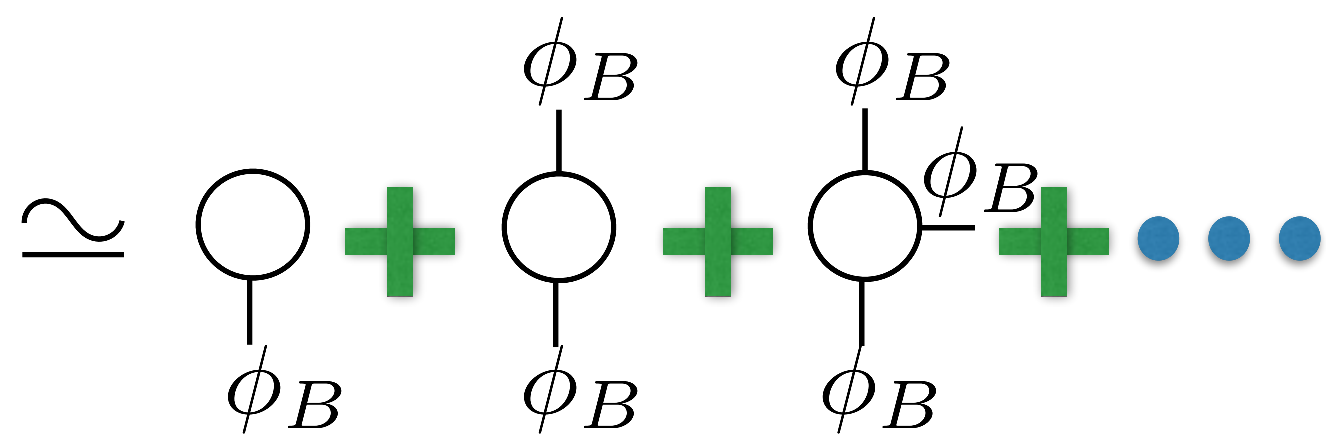
$$\gamma = A e^{-B}$$

$$A = \frac{B^2}{4\pi^2}\left(\frac{\det' S''|_{\mathrm{Bounce}}}{\det S''|_{\mathrm{False}}}\right)^{-1/2}$$



<u>Expectation</u>

cancellation of
the scale dependence
@1-loop



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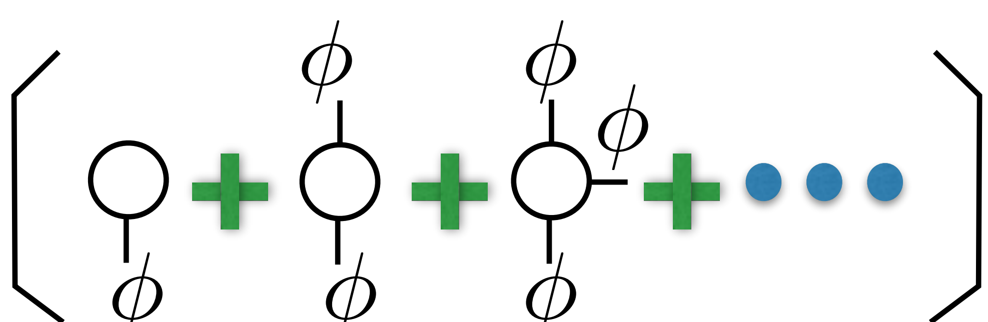


divergent part

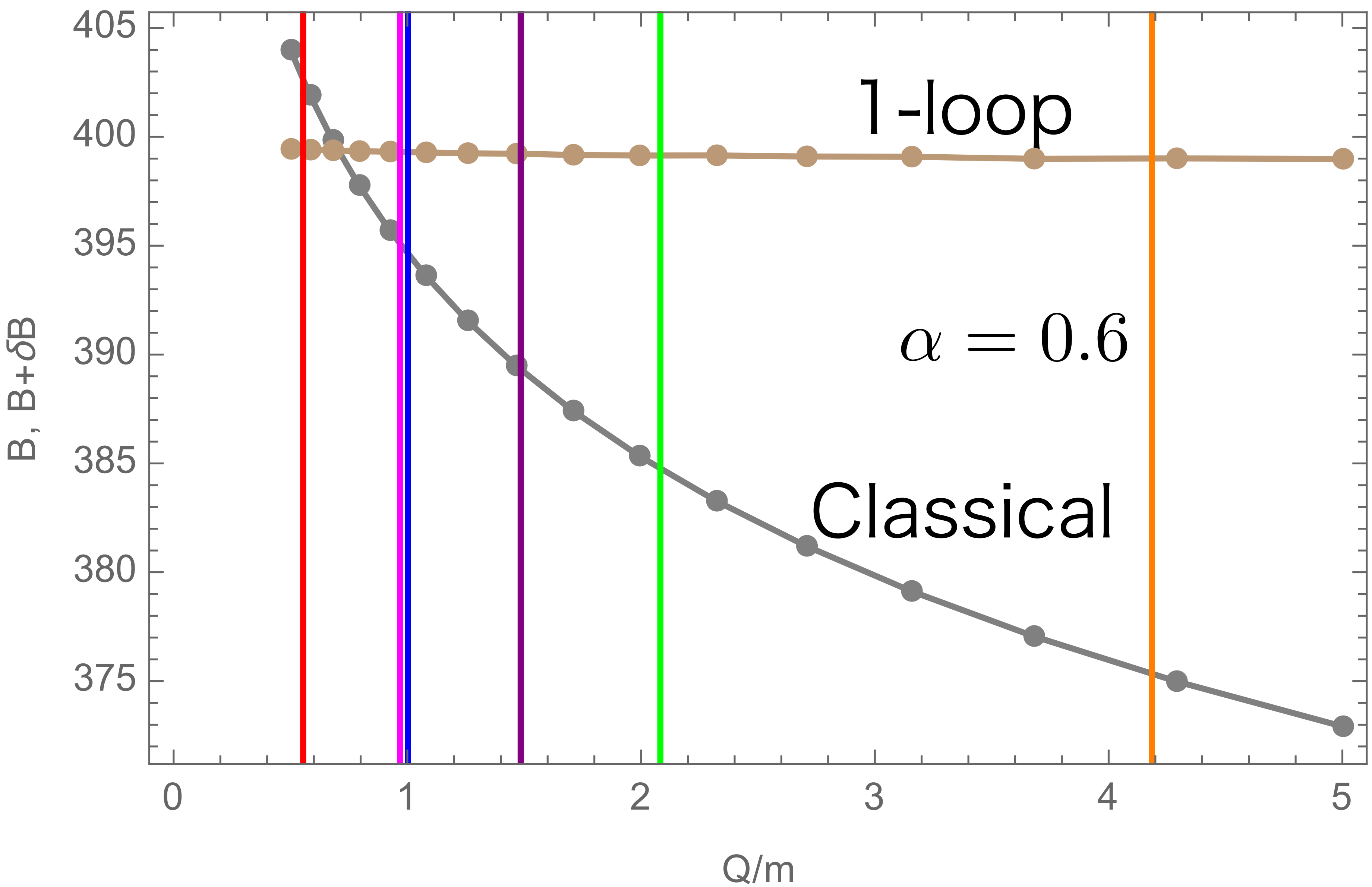
After a long,

and long

calculation,

Result

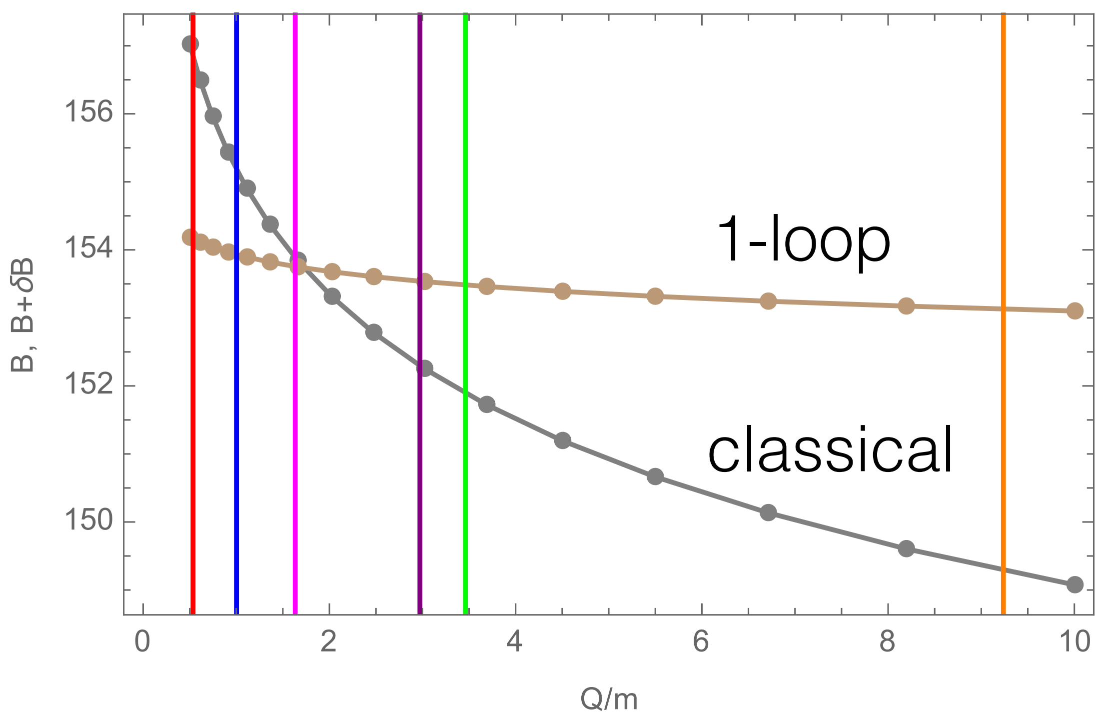
$$\gamma = Ae^{-B} \equiv m^4 e^{-B-\delta B}$$

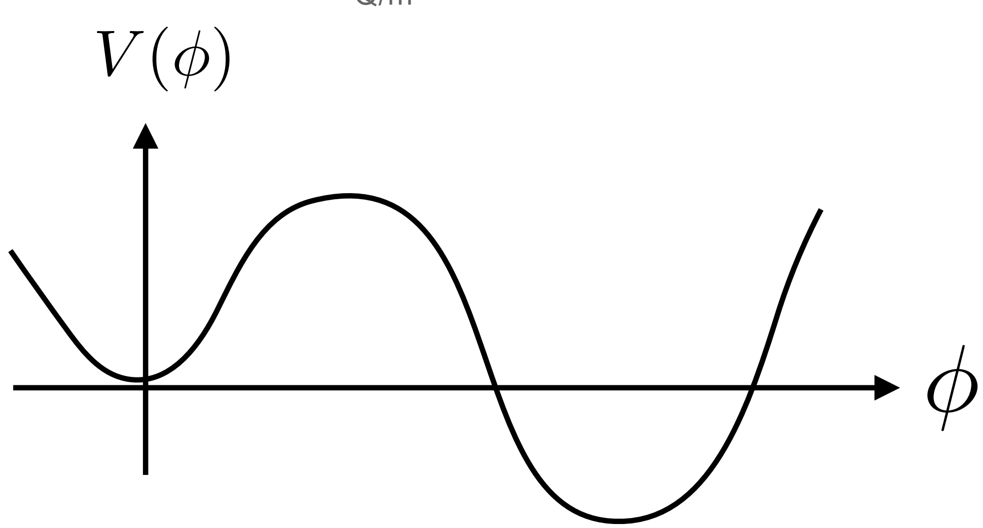


Results

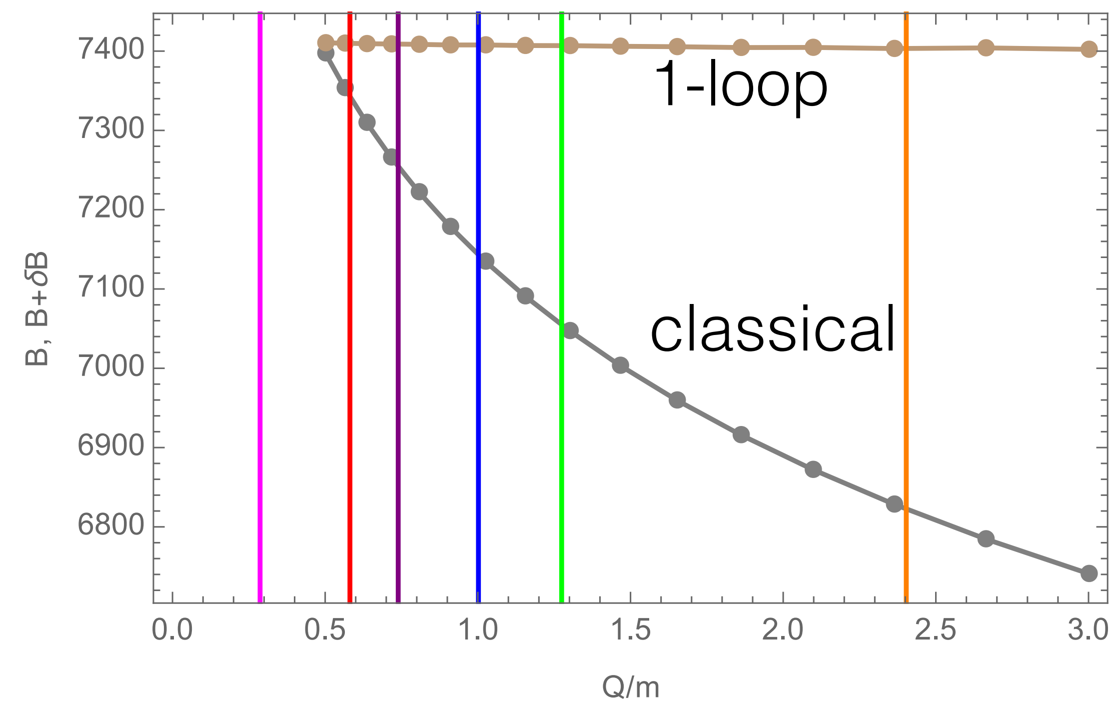
$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

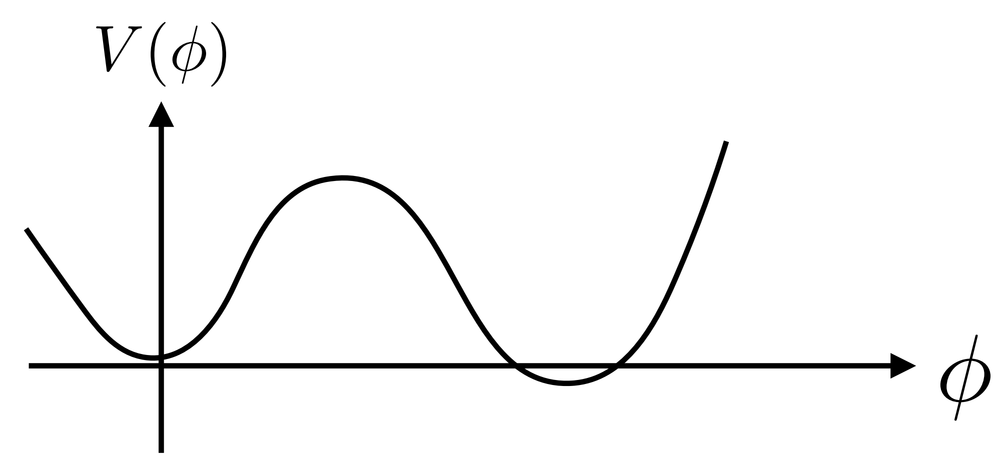
$\alpha = 0.3$





$\alpha = 0.9$





SM + stau system

Light stau

Stau can be light

$m_{\tilde{\tau}} > 103.5\mathrm{GeV}$ (LEP)

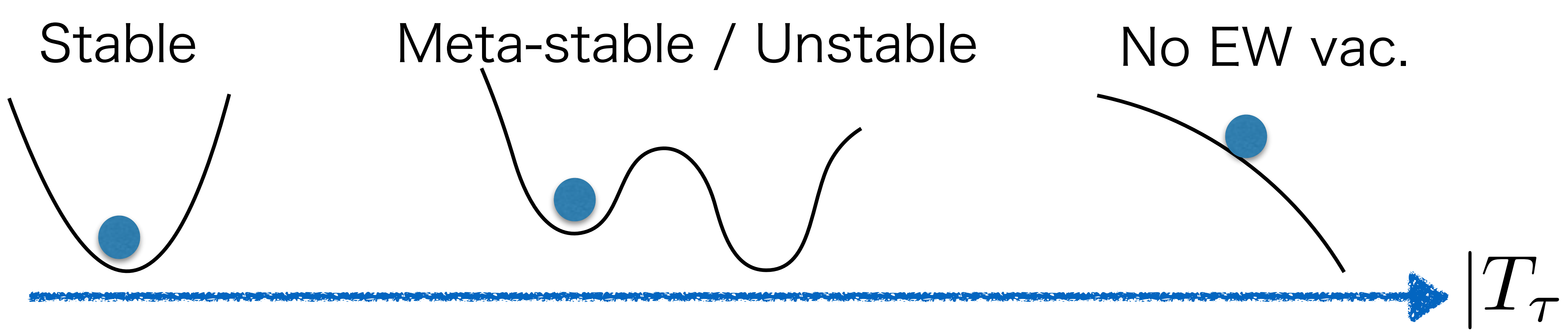
h$\gamma\gamma$ coupling, co-annihilation with bino, ⋯

But, the potential may become unstable towards the stau direction

$$V = T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + h.c.) + m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 + m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 + \cdots$$

$$T_\tau = y_\tau (A_\tau - \mu \tan\beta)$$

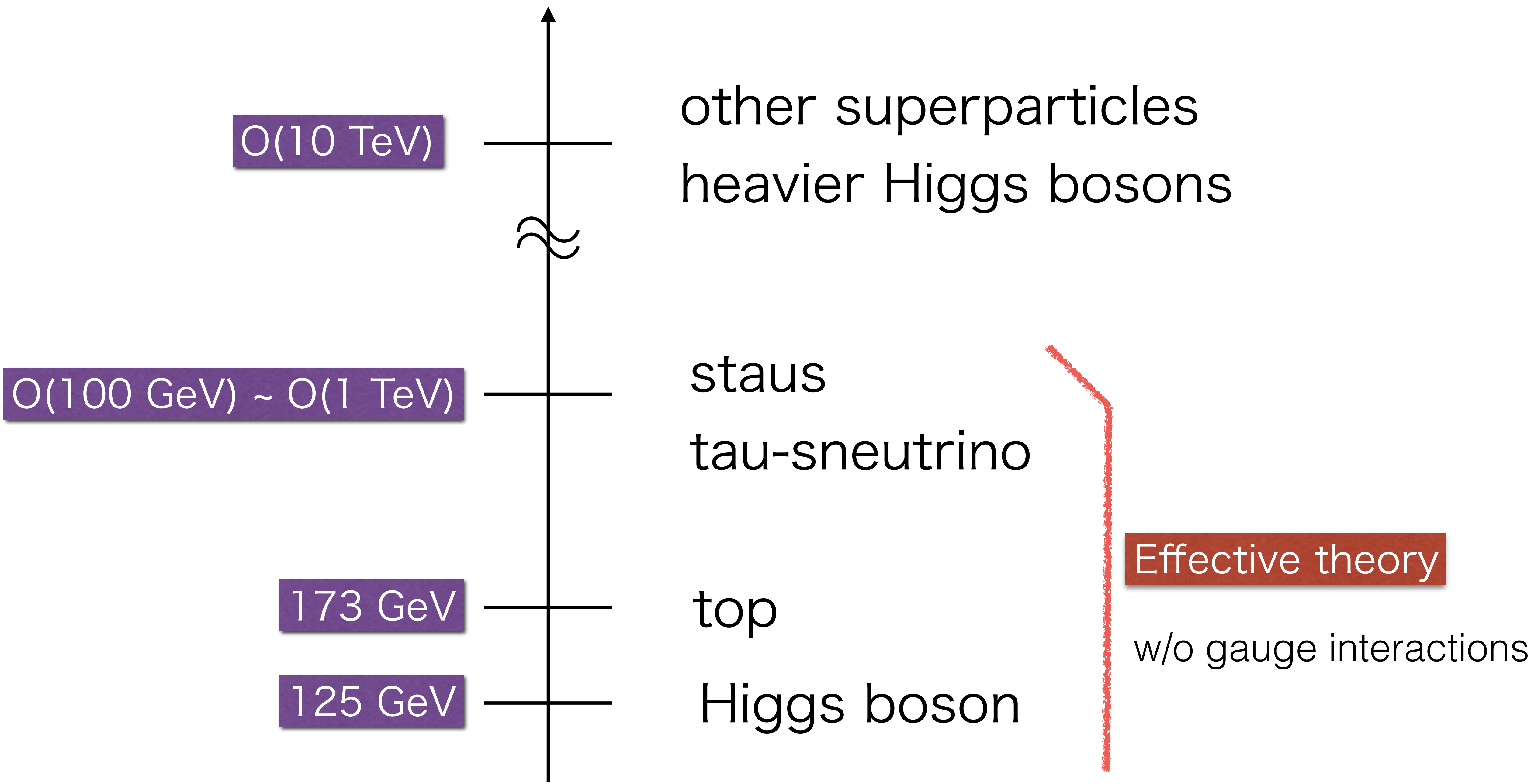
$$\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$$



Spectrum

For simplicity,

we assume only the staus are light



Effective theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm kin} - y_t(H q_L t_R^c + {\rm h.c.}) - m_H^2 |H|^2 - \frac{1}{4}\lambda_H |H|^4$$
$$- m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 - m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 - T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + {\rm h.c.}) - \frac{1}{4}\kappa^{(1)} |\tilde{\ell}_L|^4 - \frac{1}{4}\kappa^{(2)} |\tilde{\tau}_R|^4$$
$$- \frac{1}{4}\lambda^{(1)} |H|^2 |\tilde{\ell}_L|^2 - \frac{1}{4}\lambda^{(2)} |H^\dagger \tilde{\ell}_L|^2 - \frac{1}{4}\lambda^{(3)} |H|^2 |\tilde{\tau}_R|^2 - \frac{1}{4}\kappa^{(3)} |\tilde{\ell}_L|^2 |\tilde{\tau}_R|^2,$$

Boundary conditions

EW scale

$$y_t = \frac{M_t}{v},$$
$$m_H^2(M_t) = -\frac{1}{2} M_h^2,$$
$$\lambda_H(M_h) = \frac{M_h^2}{2v^2},$$

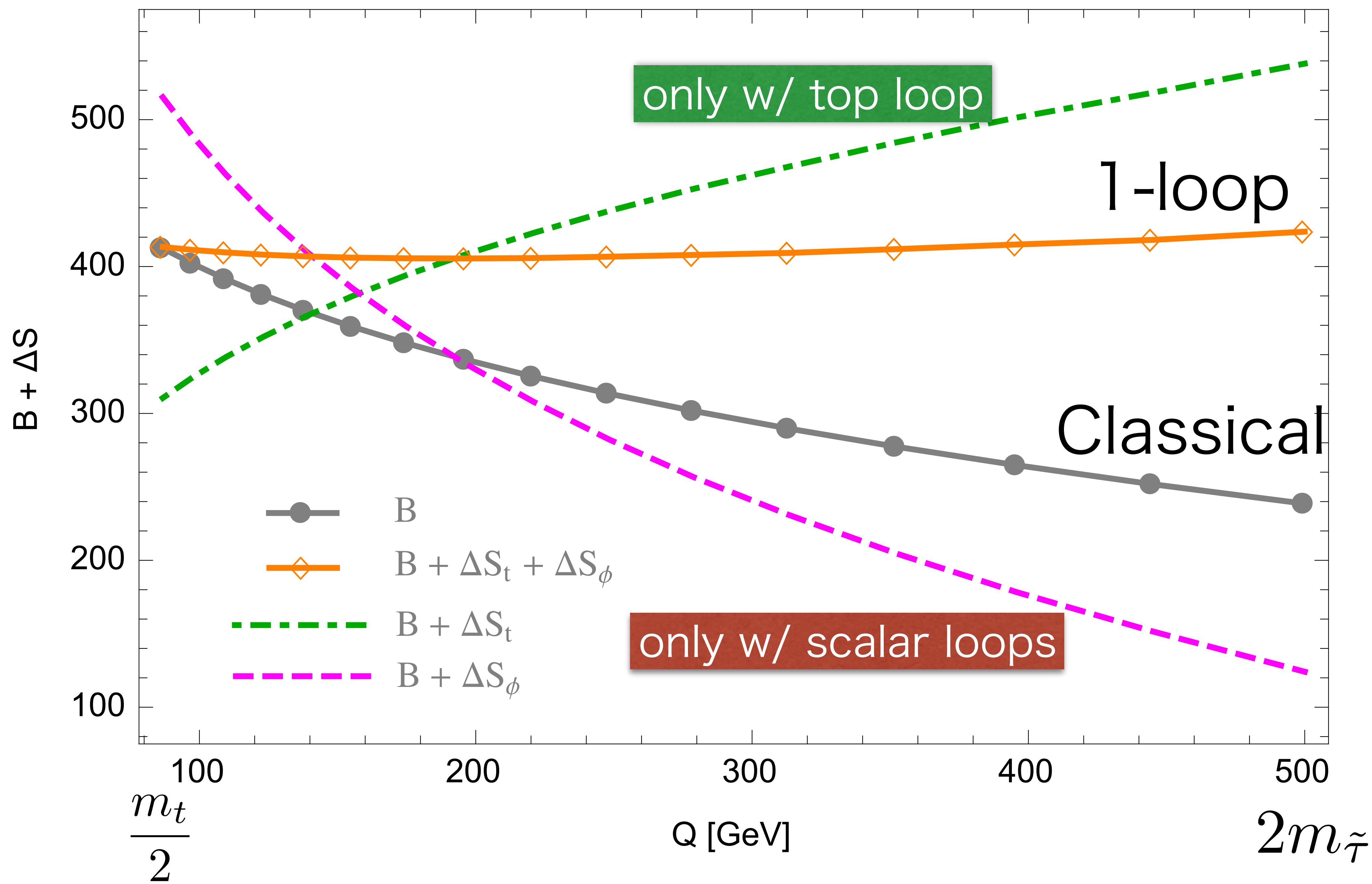
stau mass

$$m_{\tilde{\tau}} \equiv m_{\tilde{\ell}_L} = m_{\tilde{\tau}_R} = 250\,{\rm GeV},$$
$$T_\tau = 300\,{\rm GeV}.$$

SUSY scale (10TeV)

$$\lambda^{(1)}(M_{\rm SUSY}) = (g^2 + g'^2)\cos 2\beta,$$
$$\lambda^{(2)}(M_{\rm SUSY}) = 4y_\tau^2 - 2g^2 \cos 2\beta,$$
$$\lambda^{(3)}(M_{\rm SUSY}) = 4y_\tau^2 - 2g'^2 \cos 2\beta,$$
$$\kappa^{(1)}(M_{\rm SUSY}) = \frac{1}{2}(g^2 + g'^2),$$
$$\kappa^{(2)}(M_{\rm SUSY}) = -\kappa^{(3)}(M_{\rm SUSY}) = 2g'^2,$$
$$\tan\beta = 20$$

Result



only w/ top loop
1-loop
Classical
only w/ scalar loops
B
B + ΔS$_t$ + ΔS$_\phi$
B + ΔS$_t$
B + ΔS$_\phi$
B + ΔS
500
400
300
200
100
100
200
300
400
500
Q [GeV]
$\frac{m_t}{2}$
$2m_{\tilde{\tau}}$

Summary

- The bubble nucleation rate has often been estimated without calculating the pre-exponential factor.
- This estimate involves uncertainty in the renormalization scale, which, we showed, results in O(10%) uncertainty in the exponent of the bubble nucleation rate.
- To reduce the uncertainty, we explicitly calculated the pre-exponential factor and showed that it is greatly reduced.
- Scalars and fermions have already been implemented, but the gauge bosons are now ongoing.

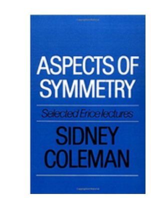
Theorem

$$\gamma = Ae^{-B}$$

function of the bounce solution

$$\ln A^{-2} = \ln \frac{\det\left[-\partial^2 + m_0^2 + \delta W\right]}{\det\left[-\partial^2 + m_0^2\right]}$$

<u>Theorem</u> (Dirichlet BC) (J. H. van Vleck, '28; R. H. Cameron and W. T. Martin, '45; …)



The ratio of the determinant of differential operators,

$$L_j = -\frac{d^2}{dx^2} + R_j(x)$$ on $$I = [0, 1]$$ w/ Diriclet BC $f(0) = 0,\ f(1) = 0$

is given by the ratio of the solutions of differential equations

$$\frac{\mathrm{Det}L_1}{\mathrm{Det}L_0} = \frac{y_1(1)}{y_0(1)} \qquad L_j y_j(x) = 0$$

$$y_j(0) = 0,\ y_j'(0) = 1$$