Particle number asymmetry induced with CP violating interaction in expanding universe

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Purpose and Introducyion

1. **Build a model generating particle and anti-particle asymmetry of universe through interactions.**
2. **Compute the time variation of asymmetry in expanding universe using quantum field theory**

Contents

1. Lagrangian of interacting model of scalars (Symmetry, CPV)

2. 2PI effective action of the model $\longrightarrow$ Equation of motion of the time dependent expectation value of the fields and Green functions.

3. Contributions to the asymmetry up to the first order of interactions

4. Summary

Model of scalars with particle number violation

N; Neutral scalar ,ϕ complex scalar

$$S = \int d^4x\sqrt{-g}(\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int.}}),$$

$$\mathcal{L}_{free} = g^{\mu\nu}\nabla_\mu\phi^*\nabla_\nu\phi + \frac{B^2}{2}(\phi^2 + \phi^{*2})$$

$$-m_\phi^2|\phi|^2 + \frac{\alpha_2}{2}(\phi^2 + \phi^{*2})R + \alpha_3|\phi|^2 R$$

$$+\frac{1}{2}(g^{\mu\nu}\nabla_\mu N\nabla_\nu N - m_N^2 N^2)$$

$$\mathcal{L}_{int.} = A\phi^2 N + A^*\phi^{*2}N + A_0|\phi|^2 N$$

The particle number related to U(1) transformation

- **U(1) transformation for the complex scalar field.**
  $\phi(x) \to e^{i\theta}\phi(x)$
- **U(1) charge** (eg.,Affleck and Dine (85) in susy)
  = particle number- anti-particle number

$$N(x^0) = \int \sqrt{-g(x)} j_0(X) d^3x$$

$$j_\mu(X) = i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi$$

Space time and initial condition

- **Space time**

  Friedman Robetson Walker universe with scale factor $a(x^0)$ of arbitary time depencdence.

  $ds^2 = dx^{02} - a^2(x^0)dx^i dx^i$,

  $R = 6(H(x^0)^2 + \frac{\ddot{a}}{a}), H(x^0) = \frac{\dot{a}}{a}$.

- **Initial conditions** Mixed state given by the density matrix $\rho(0) = \frac{e^{-\beta H}}{\mathrm{Tr} e^{-\beta \mathrm{H}}}, \beta = \frac{1}{T}$, and the initial expectation values of scalars $\bar{\phi}_i(0)$.

Rewrite complex scalar in terms of real fields.

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}},\ N \equiv \phi_3$$

$$\mathcal{L}_{free} = \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi_i \nabla_\nu \phi_i) - \frac{\tilde{m}_i^2}{2} \phi_i^2$$

Effects of non-zero B^2, α_2 which violates U(1)

Non-degeneracy of two real scalars;

$$\tilde{m}_1^2 = m_\phi^2 - \alpha_3 R - B^2 - \alpha_2 R$$

$$\tilde{m}_2^2 = m_\phi^2 - \alpha_3 R + B^2 + \alpha_2 R$$

Interaction in terms of real scalars

$$\mathcal{L}_{int.} = \sum_{ijk=1}^{3} \frac{A_{ijk}}{3} \phi_i \phi_j \phi_k$$

$A_{113} = \frac{A_0}{2} + \mathrm{Re.}(A)$	
$A_{223} = \frac{A_0}{2} - \mathrm{Re.}(A)$	
$A_{113} - A_{223} = 2\mathrm{Re.}(A)$	U(1) violation
$A_{123} = -\mathrm{Im.}(A)$	U(1), CP violation
$B^2\phi^2, \alpha_2 R\phi^2$	U(1) violation

Current expection value in terms of real fields

$$j_\mu = \frac{1}{2}\left(\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \phi_1 \overleftrightarrow{\partial}_\mu \phi_2\right)$$

Expectaion value with $\rho(0)$ (initial density matrix); mixed state.

$$\langle j_\mu(x)\rangle = \mathrm{Tr}[j_\mu(x)\rho(0)]$$
$$= \lim_{y\to x}\left(\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial y^\mu}\right)\mathrm{Re.}[G_{12}(x,y)]$$
$$+ \mathrm{Re.}\left(\bar{\phi}_2^* \overleftrightarrow{\partial}_\mu \bar{\phi}_1\right)$$

$G_{ij}(x,y)$ and $\bar{\phi}_i$ are obtained from effective action.

Effective action for Green functions and field expectation

$$\Gamma[G,\bar{\phi},g] = S[\bar{\phi},g] + \frac{i}{2}\mathrm{TrLnG}^{-1}$$

$$+\frac{1}{2}\int d^4x\int d^4y\frac{\delta^2 S}{\delta\bar{\phi}^a_i(x)\delta\bar{\phi}^b_j(y)}G^{ab}_{ij}(x,y)$$

$$+\frac{i}{3}D_{abc}A_{ijk}\int\int d^4xd^4y\sqrt{-g_x}\sqrt{-g_y}$$

$$[G^{aa'}_{ii'}(x,y)G^{bb'}_{jj'}(x,y)G^{cc'}_{kk'}(x,y)]$$

$D_{a'b'c'}A_{i'j'k'} \leftrightarrow$ Γ_Q =2 PI graph in next page

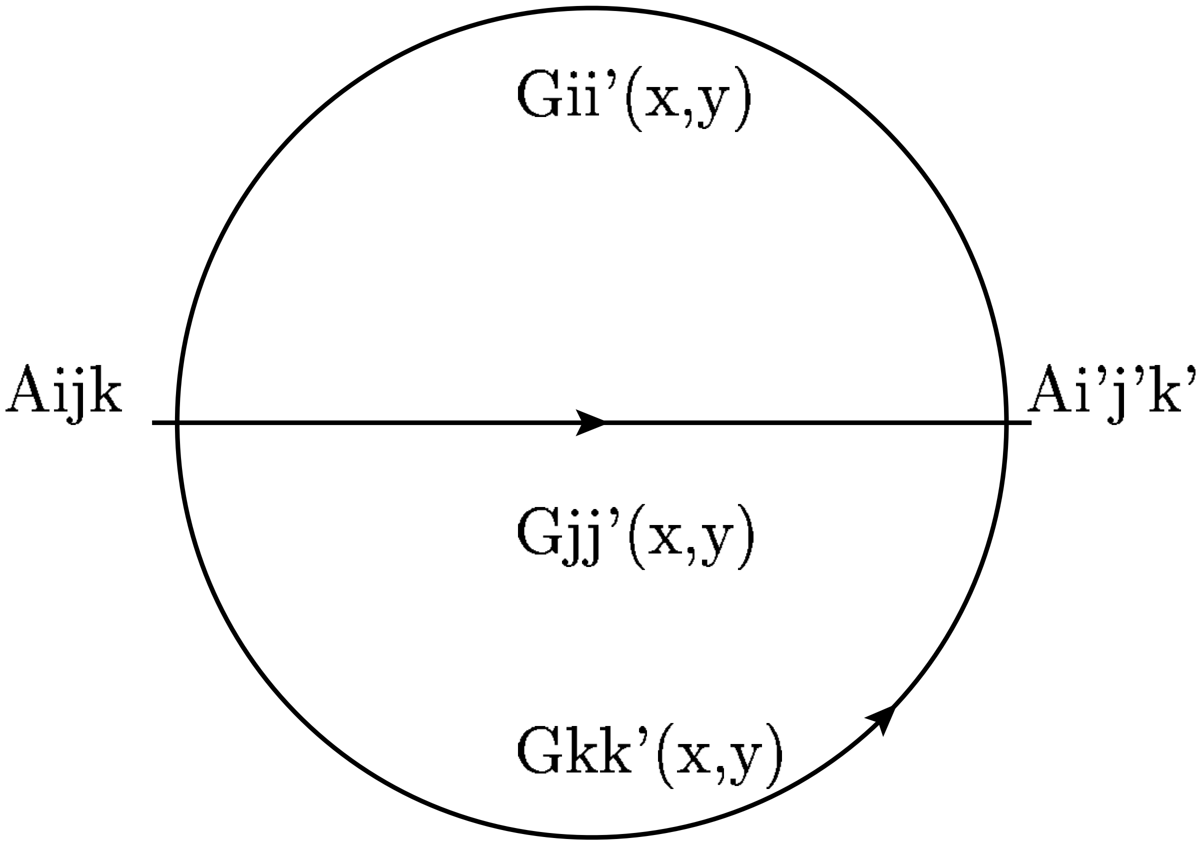

Figure 1: **2 PI graph; Γ_Q**

Free part contribution

Free part : $B^2, \alpha_2 \neq 0, A_{ijk} = 0$

Rescaled field expectation value:

$$\bar{\varphi}(x^0) \equiv \left(\frac{a_0}{a(x^0)}\right)^{\frac{3}{2}} \bar{\phi}_{\mathrm{free}}(x^0)$$

$$\left(\frac{a(x^0)}{a_0}\right)^3 \langle j_0(x^0)\rangle_{\mathrm{free}} = Re.[\bar{\varphi}_2^*(x^0)\frac{\overleftrightarrow{\partial}}{\partial x^0}\bar{\varphi}_1(x^0)]$$

Free part contribution

$$= \bar{\phi}_1(0)\bar{\phi}_2(0)[K_2'(x^0,0)\frac{\overleftrightarrow{\partial}}{\partial x^0}K_1'(x^0,0)$$

$$+\,\omega_{1,0}\omega_{2,0}\tanh\frac{\beta\omega_{1,0}}{2}\tanh\frac{\beta\omega_{2,0}}{2}$$

$$\times\, K_2(x^0,0)\frac{\overleftrightarrow{\partial}}{\partial x^0}K_1(x^0,0)]$$

Depends on initial values of $\bar{\phi}_1(0)\bar{\phi}_2(0)$.

Under CP; $\rightarrow$ $-\bar{\phi}_1(0)\bar{\phi}_2(0)$.

Time dependence of the free part

- **Oscillation behaviour** for non degenerate ϕ_1 and ϕ_2.
- **In the limit for vanishing particle number** violating mass terms; $B^2, \alpha_2 \longrightarrow 0, K_1 = K_2$ and time dependent part vanishes.

  In the limit of $H = 0$,

$$K_{1,2}(x^0, 0) = \frac{\sin m_{1,2} x^0}{m_{1,2}}, m_{1,2} = \sqrt{m_\phi^2 \mp B^2}$$

O(A) contribution to the current(I)

$$\left(\frac{a(x_0)}{a_0}\right)^3 \langle j_0(x^0)\rangle_{O(A)} =$$

$$+ Re.[\bar{\varphi}^*_{2free}(x^0)\frac{\overleftrightarrow{\partial}}{\partial x^0}\bar{\varphi}_{1O(A)}(x^0)]$$

$$- Re.[\bar{\varphi}^*_{1free}(x^0)\frac{\overleftrightarrow{\partial}}{\partial x^0}\bar{\varphi}_{2O(A)}(x^0)]$$

$$\bar{\varphi}_2^*(x^0)\overleftrightarrow{\partial_{x^0}}\bar{\varphi}_{1O(A)}(x^0) = \int_0^{x^0} dt\bar{\varphi}_2^*(x^0)\overleftrightarrow{\partial_{x^0}}K_1(x^0,t)$$

$$(A_{113}(t)\bar{\varphi}_1(t) + A_{123}(t)\bar{\varphi}_2(t))\bar{\varphi}_3(t),$$

$$-\bar{\varphi}_1^*(x^0)\overleftrightarrow{\partial_{x^0}}\bar{\varphi}_{2O(A)}(x^0) = -\int_0^{x^0} dt\bar{\varphi}_1^*(x^0)\overleftrightarrow{\partial_{x^0}}K_2(x^0,t)$$

$$(A_{213}(t)\bar{\varphi}_1(t) + A_{223}(t)\bar{\varphi}_2(t))\bar{\varphi}_3(t).$$

O(A) contribution to the current (I)

(1) The contribution to the asymmetry with A_{123}(CPV U(1)) is propotional to $\phi_2(0)^2\phi_3(0)$ and $\phi_1(0)^2\phi_3(0)$.

(2) The contribution to the asymmetry with A_{113}, A_{223} is propotinal to $\phi_1(0)\phi_2(0)\phi_3(0)$.

(3) The asymmetry depends on the initial conditions such as $\phi_1(0)^2 - \phi_2(0)^2$ and the sign of $\phi_1(0)\phi_2(0)$ etc.

O(A) contribution to the current(II) through Green function

$$\langle j_0(x^0)\rangle =$$

$$\lim_{y^0\to x^0}\int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial}{\partial x^0}-\frac{\partial}{\partial y^0}\right)\mathrm{Re.}[G_{12O(A)}(x^0,y^0,\mathrm{k})]$$

$$\langle j_0(x^0)\rangle_{O(A)}$$

$$= A_{123}\bar{\phi}_3(0)\int \frac{k^2 dk}{2\pi^2}\left(\frac{1}{\omega_2(k)} - \frac{1}{\omega_1(k)}\right)$$

$$[\frac{\sin\frac{\omega_1(k)+\omega_2(k)+\omega_3}{2}x^0 \cos\frac{\omega_1(k)+\omega_2(k)-\omega_3}{2}x^0}{\omega_1(k)+\omega_2(k)+\omega_3})$$

$$\times(1+\tanh\frac{\omega_3}{2T} + n_1(k) + n_2(k))) + (\omega_3 \to -\omega_3)]$$

$$+ otherterms.$$

$$n(k) = \frac{1}{e^{\beta\omega(k)-1}}.$$

The analytic expression shows

- **The starting with the ininitial condition $< j^0(x^0 = 0) >= 0$, the particle number asymmetry is created through interactions.**

- **The asymmetry is proportional to CP violating coupling A_{123}, the mass difference $m_1 \neq m_2$ (the particle number violating mass term) and the initial expectation value of the neutral scalor $\bar{\phi}_3(x^0 = 0)$.**

Summary of our reserach

- In interacting model, 2 PI effective action $\Gamma[G, \bar{\phi}]$ is obtained. Schwinger Dyson equation for Green functions G and expectation value $\bar{\phi}$ for the fields are obtained in the form of integral equation.
- The integral equations are iteratively solved by treating interaction A_{ijk} is small.
- The current for the particle and anti-particle asymmetry is given up to the first order of A.