2016年3月25日
松江素粒子物理学研究会

複合スカラー場によるインフレーション

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Astr. Space Sci. 360 (2015) 67
[arXiv:1509.03738]

本日の話題

- 複合スカラー場の模型
- インフレーションと素粒子模型
- 宇宙背景輻射の揺らぎ
- インフレーションからの離脱
- まとめと今後の課題

Gauged Nambu-Jona-Lasinio model

複合スカラー場の模型

Gauged Nambu-Jona-Lasinio model

$SU(N) \otimes \mathcal{G}$ gauge theory with N_f fermion flavors

$\downarrow$ Strong enough

Four-fermion interaction

- Lagrangian density

$$\mathcal{L}_{gNJL} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi} i \hat{\not{D}} \psi + \frac{16\pi^2 g_4}{8 N_f N_c \Lambda^2} \left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i \gamma_5 \tau^a \psi\right)^2 \right]$$

Auxiliary field method

- Equivalent Lagrangian density

$$\mathcal{L}_{aux} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi - \frac{2N_fN_c\Lambda^2}{16\pi^2 g_4}\left(\sigma^2 + \pi^{a\,2}\right)$$

with

$$\sigma = -\frac{16\pi^2 g_4}{4N_fN_c\Lambda^2}\bar{\psi}\psi, \quad \pi^a = -\frac{16\pi^2 g_4}{4N_fN_c\Lambda^2}\bar{\psi}i\gamma_5\tau^a\psi$$

Auxiliary field method

- Equivalent Lagrangian density

$$\mathcal{L}_{aux} \quad = \quad \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi$$
$$-\frac{2N_f N_c \Lambda^2}{16\pi^2 g_4}\left(\sigma^2 + \pi^{a\,2}\right)$$

- Gauged Higgs-Yukawa Lagrangian

$$\mathcal{L}_{gHY} = \quad \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - y\sigma - yi\gamma_5\tau^a\pi^a\right)\psi$$
$$-\frac{1}{2}m^2(\sigma^2 + \pi^a\pi^a) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a$$
$$-\frac{1}{2}\xi R(\sigma^2 + \pi^a\pi^a) - \frac{\lambda}{4}(\sigma^2 + \pi^a\pi^a)^2$$

Auxiliary field method

- Equivalent Lagrangian density

$$\mathcal{L}_{aux} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi - \frac{2N_fN_c\Lambda^2}{16\pi^2 g_4}\left(\sigma^2 + \pi^{a2}\right)$$

- Gauged Higgs-Yukawa Lagrangian

$$\mathcal{L}_{gHY} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - y\sigma - yi\gamma_5\tau^a\pi^a\right)\psi - \frac{1}{2}m^2(\sigma^2 + \pi^a\pi^a) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a - \frac{1}{2}\xi R(\sigma^2 + \pi^a\pi^a) - \frac{\lambda}{4}(\sigma^2 + \pi^a\pi^a)^2$$

Conventional normalization

- Transforming the fields in the gauged Higgs-Yukawa Lagrangian

$$\sigma \to \sigma/y,\ \pi^a \to \pi^a/y$$

we get

$$\begin{aligned}
\mathcal{L}_{gHY} &= \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi \\
&\quad -\frac{1}{2}\frac{m^2}{y^2}(\sigma^2 + \pi^a\pi^a) + \frac{1}{2y^2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2y^2}\partial_\mu\pi^a\partial^\mu\pi^a \\
&\quad -\frac{\xi}{2y^2}R(\sigma^2 + \pi^a\pi^a) - \frac{\lambda}{4y^4}(\sigma^2 + \pi^a\pi^a)^2
\end{aligned}$$

W. A. Bardeen, C. Hill & M. Lindner, Phys. Rev. D41 (1990) 1647
C. T. Hill & D. S. Salopek, Annals Phys. 213 (1992) 21

Compositeness condition

- We set the following conditions at the composite scale Λ

$$\frac{1}{y^2(\Lambda)} = 0,\ \frac{\lambda(\Lambda)}{y^4(\Lambda)} = 0,\ \xi(\Lambda) = \frac{1}{6},\ \frac{m^2(\Lambda)}{y^2(\Lambda)} = \frac{2a}{16\pi^2}\Lambda^2\left(\frac{1}{g_4} - \frac{1}{\Omega(\Lambda)}\right)$$

$$\begin{aligned}
\mathcal{L}_{gHY} &= \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi \\
&\quad -\frac{1}{2}\frac{m^2}{y^2}(\sigma^2 + \pi^a\pi^a) + \cancel{\frac{1}{2y^2}}\partial_\mu\sigma\partial^\mu\sigma + \cancel{\frac{1}{2y^2}}\partial_\mu\pi^a\partial^\mu\pi^a \\
&\quad -\cancel{\frac{\xi}{2y^2}}R(\sigma^2 + \pi^a\pi^a) - \cancel{\frac{\lambda}{4y^4}}(\sigma^2 + \pi^a\pi^a)^2
\end{aligned}$$

σ, π^a: composite scalar fields

W. A. Bardeen, C. Hill & M. Lindner, Phys. Rev. D41 (1990) 1647
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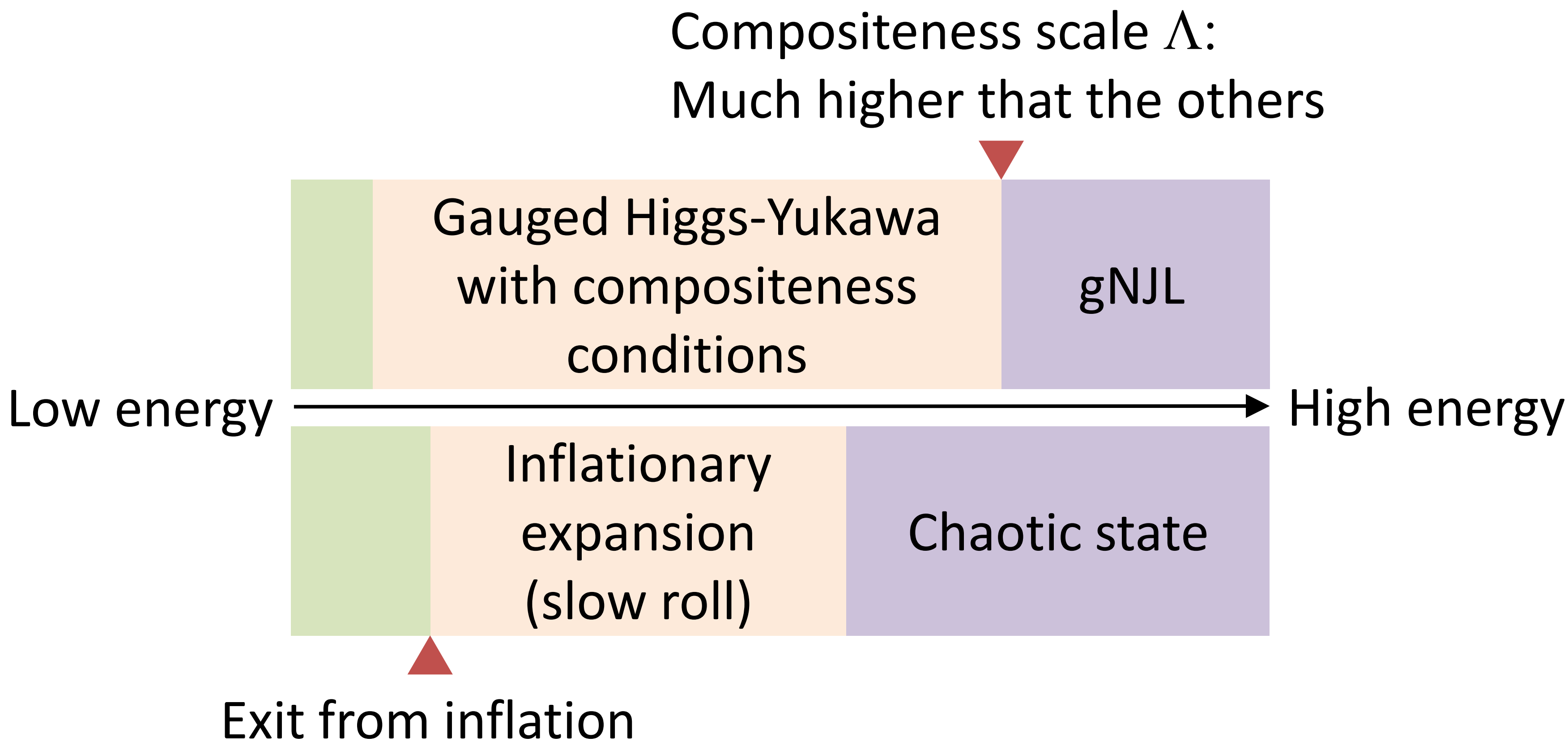
Assumptions of our analysis



Assumptions of our analysis

- We neglect the running of the SU(Nc) gauge coupling, α.
- We take the limit $\Lambda \to \infty$.
- We omit higher order terms in R.
- Only the field, σ, contributes the inflationary expansion.

M. Harada, Y. Kikukawa, T. Kugo, H. Nakano, Prog. Theor. Phys. 92 (1994) 1161
B. Geyer and S. D. Odintsov, Phys. Lett. B376 (1996a) 260

RG improved effective potential

- RG improved effective potential at the scale μ

$$V(\sigma,\mu) = \frac{B(\mu)}{2}\sigma^2(\mu) + \frac{C(\mu)}{4}[\sigma^2(\mu)]^{2/(2-w)} + \frac{D(\mu)R}{12}[\sigma^2(\mu)]^{1/(2-w)}$$

with

$$B(\mu) \equiv \frac{\alpha}{\alpha_c}\left(\frac{1}{g_{4R}(\mu)} - \frac{1}{g^*_{4R}}\right)\mu^2, \qquad \omega = 1 - \frac{\alpha}{2\alpha_c},\ \alpha_c = \frac{2\pi N_c}{3(N_c^2-1)}$$

$$C(\mu) \equiv \frac{N_f N_c}{4\pi^2}\left(\frac{4\pi^2}{N_f N_c \mu^2}\frac{\alpha}{\alpha_c}\right)^{2/(2-\omega)}\left(\frac{3}{2} + \frac{\alpha_c}{\alpha}\right)\mu^4,$$

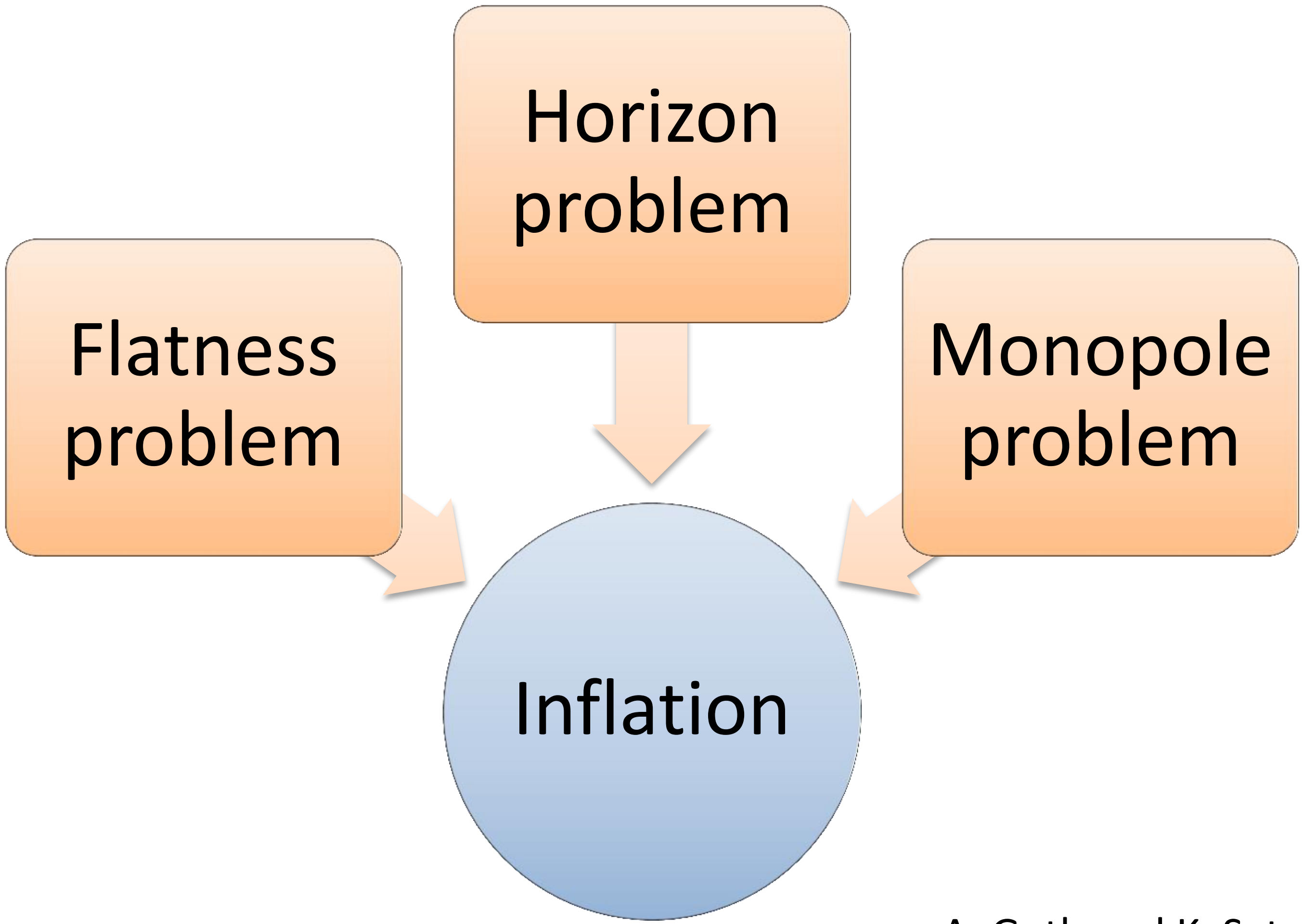
$$D(\mu) \equiv \frac{N_f N_C}{4\pi^2}\left(\frac{4\pi^2}{N_f N_c \mu^2}\frac{\alpha}{\alpha_c}\right)^{1/(2-\omega)}\left(\frac{1}{2} + \frac{\alpha_c}{\alpha}\right)\mu^2$$

B. Geyer and S. D. Odintsov, Phys. Lett. B376 (1996a) 260

Inflationary expanding universe

インフレーションと素粒子模型

Inflationary expansion of early universe



A. Guth and K. Sato, 1981

Evolution of universe

- Locally flat Friedman-Robertson-Walker universe

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

- Equation of motion for σ

$$\ddot{\sigma} + 3H\dot{\sigma} = -\frac{\partial V}{\partial \sigma} \ , H \equiv \frac{\dot{a}}{a}$$

- Friedman equation

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\sigma}^2 + V\right)$$

Quasi-de Sitter solution

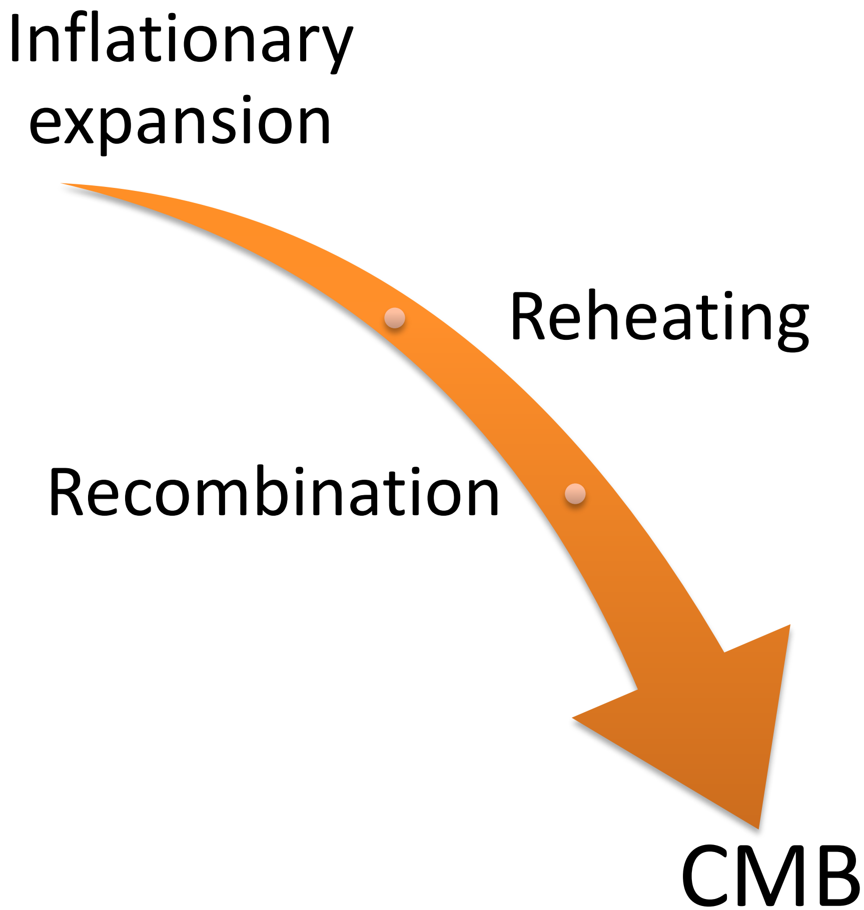
- Slow roll approximation

$$\dot{\sigma}^2 \ll V, \ \ddot{\sigma} \ll \left| \frac{\partial V}{\partial \sigma} \right|$$

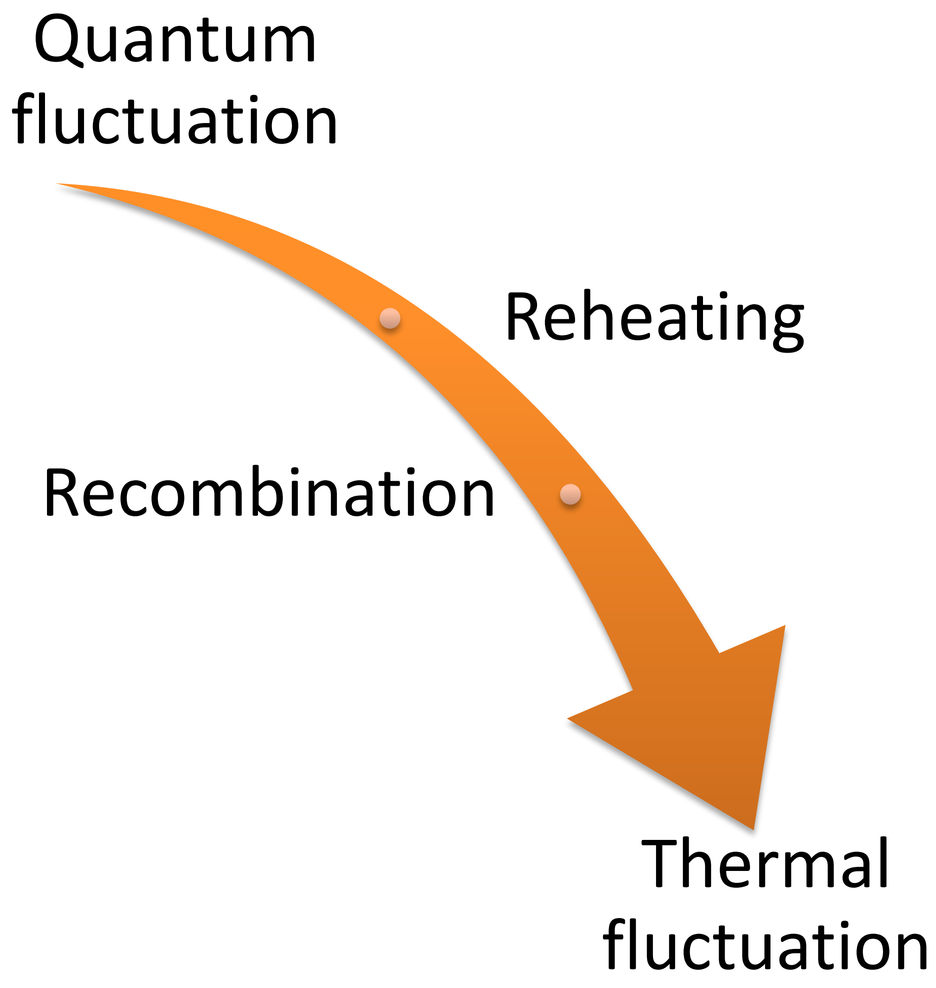
- Approximate solution

$$a(t+\Delta t) \sim x a(t) e^{\sqrt{\frac{V}{3}}\Delta t} + (1-x) a(t) e^{-\sqrt{\frac{V}{3}}\Delta t}$$

Cosmic microwave background (CMB)



CMB fluctuations



CMB fluctuations

宇宙背景輻射の揺らぎ

CMB fluctuation in the slow roll scenario

Weyl transformation and field redefinition to the Einstein frame

The exit from the inflation is found by evaluating the slow roll parameter, $\varepsilon>1$.

The value of σ at the horizon crossing is fixed to generate a suitable e-folding number, N=50-60.

Density fluctuation, spectral index, tensor-to-scalar ratio, running of the spectral index

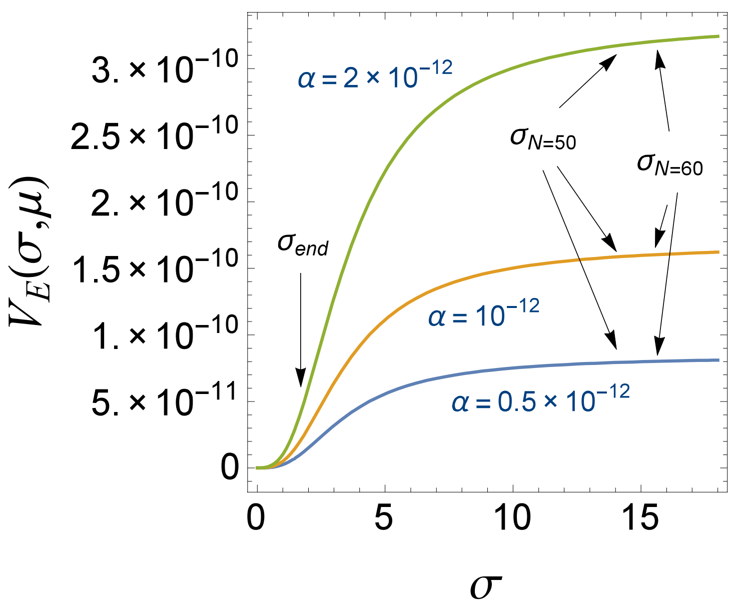
Numerical calculations

- Model parameters
  - # of colors: N_c
  - # of flavors: N_f
  - SU(Nc) gauge coupling: α
  - Four-fermion coupling: g_{4R}
  - Energy scale: μ

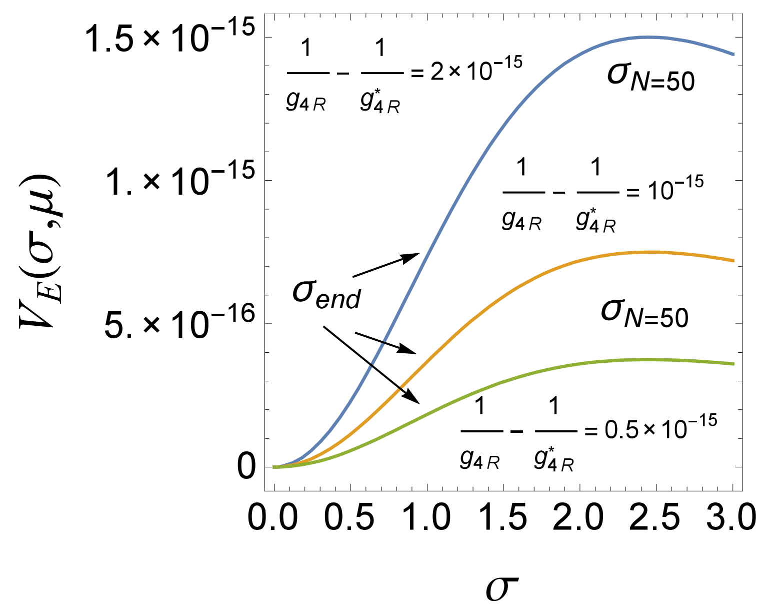
All the mass scale is normalized by the Planck scale.

RG improved effective potential

Gauged NJL model ($N_f N_c < 24\pi^2$)



NJL limit of the model



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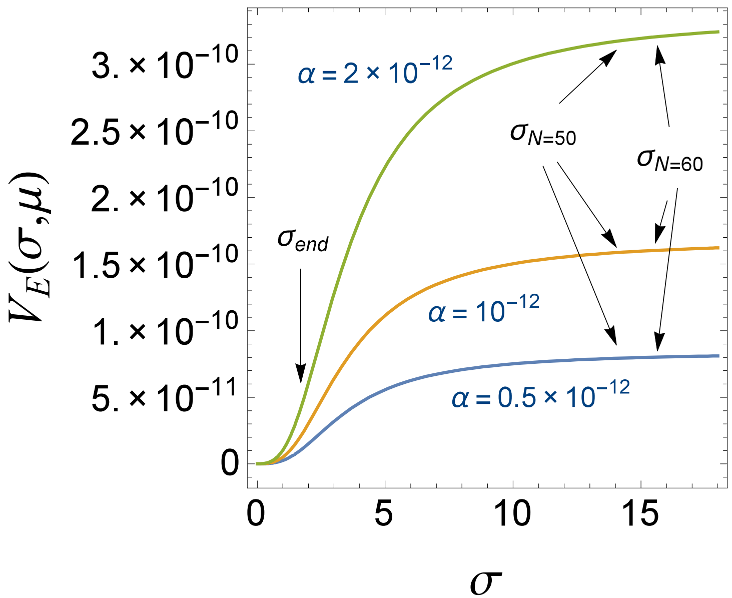

RG improved effective potential

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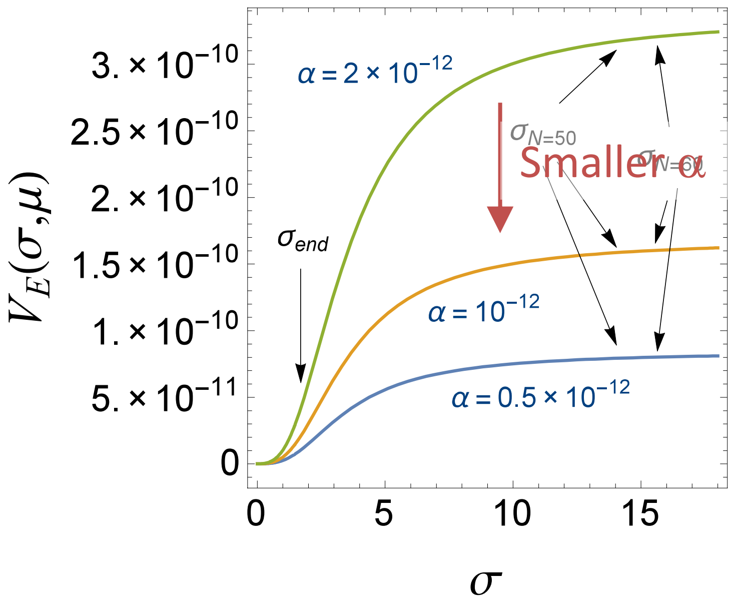

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Numerical calculations

- Model parameters
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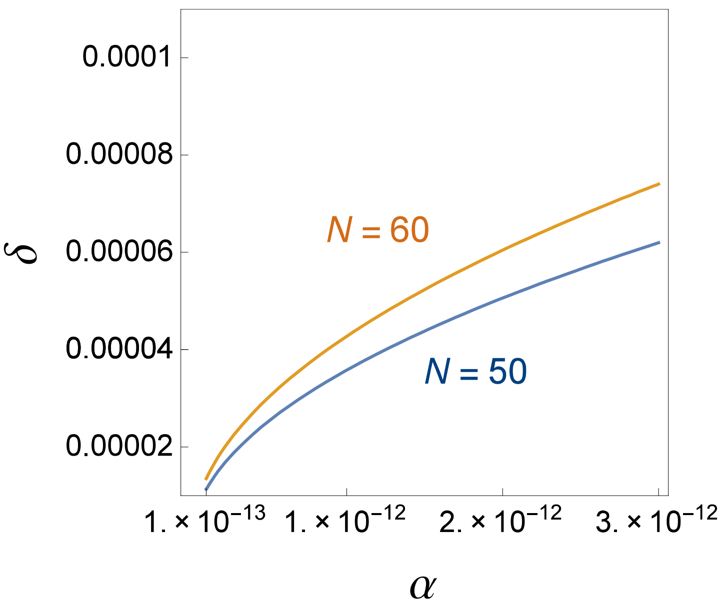
All the mass scale is normalized by the Planck scale.

Density fluctuation δ

Observed value: $\delta \sim 4.93 \times 10^{-5}$



α	δ	n_s	r	α_s
10^{-12}	3.58×10^{-5}	0.961	0.0084	-0.00077
10^{-11}	1.13×10^{-4}	0.961	0.0084	-0.00077
10^{-10}	3.58×10^{-4}	0.961	0.0084	-0.00077

$N_f = 1,\ N_c = 10,\ 1/g_{4R} - 1/g_{4R}^* = 1,\ \mu = 1$ and $N = 50$

Numerical calculations

- Model parameters
  - # of colors: N_c
  - # of flavors: N_f
  - SU(Nc) gauge coupling: $\alpha = 10^{-12}$
  - Four-fermion coupling: g_{4R}
  - Energy scale: μ

All the mass scale is normalized by the Planck scale.

Numerical calculations

- Model parameters
  - # of colors: N_c
  - # of flavors: N_f
  - SU(Nc) gauge coupling: $\alpha = 10^{-12}$
  - Four-fermion coupling: g_{4R}
  - Energy scale: μ

All the mass scale is normalized by the Planck scale.

spectral index n_s, tensor-to-scalar ratio r & running of the spectral index α_s

Observed value: $n_s = 0.9655 \pm 0.0062,\ r < 0.10,$
(Planck 2015) $\alpha_s = -0.0084 \pm 0.0082$

μ	$1/g_{4R} - 1/g_{4R}^*$	δ	n_s	r	α_s
10^{-8}	1	3.59×10^{-5}	0.961	0.0083	-0.00076
10^{-4}	1	3.59×10^{-5}	0.961	0.0083	-0.00076
1	1	3.58×10^{-5}	0.961	0.0084	-0.00077
1	10^{-4}	3.59×10^{-5}	0.961	0.0083	-0.00076
1	10^{-8}	3.59×10^{-5}	0.961	0.0083	-0.00076

$N_f = 1,\ N_c = 10,\ \alpha = 10^{-12}$ and $N = 50$

Exit from Inflationary

インフレーションからの離脱

End of slow roll era

- Deceleration parameter

  $$q \equiv -\frac{\ddot{a}}{H^2 a} \sim \epsilon - 1$$

  becomes positive for $\varepsilon > 1$.
- Duration of the inflation

$$t_{duration} = \int_{\sigma_N}^{\sigma_{end}} \left(\frac{\partial \sigma}{\partial \varphi}\right)^2 \frac{\sqrt{3V_E}}{\partial V_E / \partial \sigma} d\sigma$$

$$= (7 \sim 8) \times 10^6 M_p^{-1} \sim 10^{-36}\,\mathrm{s}$$

Exit from the de-Sitter era

- Perturbation around the quasi de Sitter solution

$$H = H_{dS} + \Delta H(t), \quad H_{dS} \equiv V_E/3$$

- From the equation of motion and the Friedman equation, we obtain

$$\Delta H(t) = A \sinh\left(6\sqrt{\frac{V}{3}}t\right)$$

➡ the quasi de Sitter solution is unstable.

K. Bamba, R. Myrzakulov, S. D. Odintsov, L. Sebastiani, Phys. Rev. D90 (2014) 043505

Summary and interesting extensions

まとめと今後の課題

Summary

- Inflationary expanding universe has been investigated in a composite Higgs model, the gauged NJL model.
- CMB fluctuations are calculated under the slow roll approximation.
- The gauge coupling α should be the order $O(10^{-12})$ to generate a suitable density fluctuation.
- We obtain a consistent spectral index, tensor-to-scalar ratio, running of the spectral index with the Planck 2015 data.

Interesting extensions

- Lower compositeness scale (in progress)
  - Larger parameter dependences
- Meson and baryon like composite fields
  - Multi fields model of inflation
- Coupling with the SM fields
  - Preheating and reheating era
- Dark matter candidate